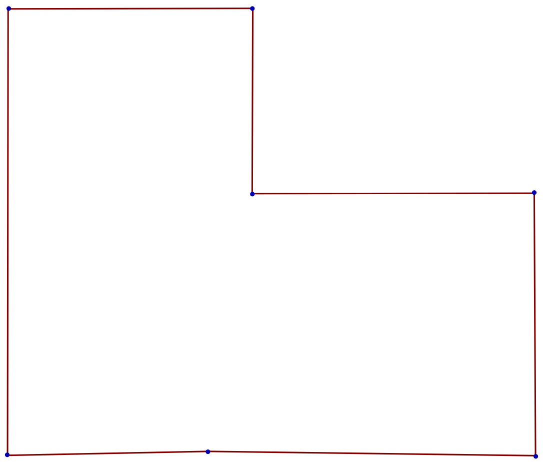
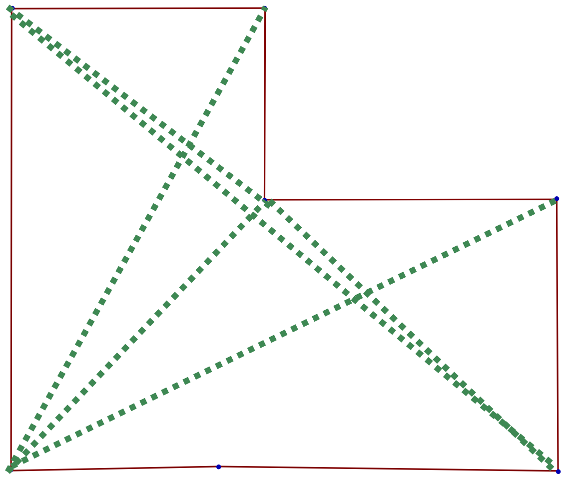
Visibility Graphs of Staircase Polygons



G



Vis (G)

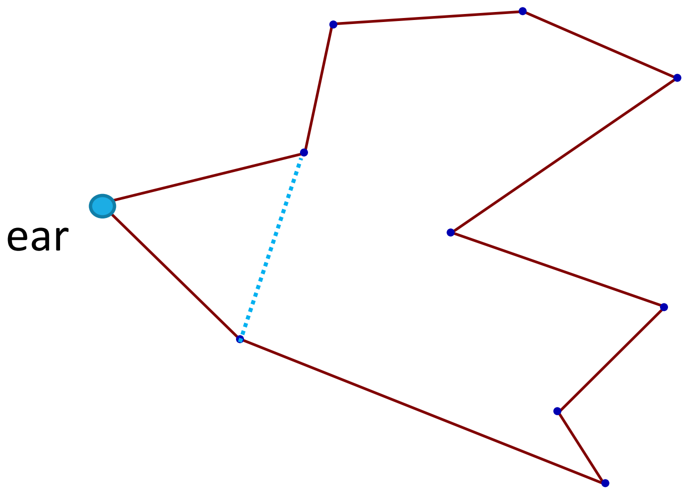
Mentor: Prof. James Abello

Work supported by NSF grant CCF-1559855.

Yulia Alexandr

Terminology

- We consider a simple non-degenerate collection of points in the plane that produces a <u>polygon.</u>



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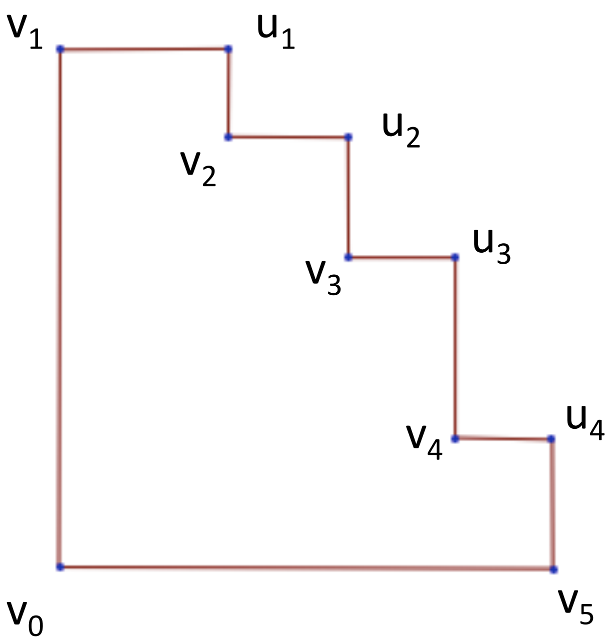
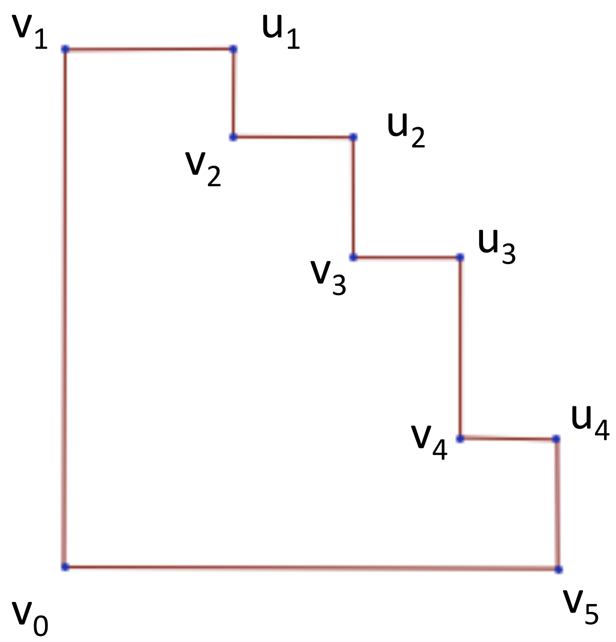
- We consider a simple non-degenerate collection of points in the plane that produces a polygon.

- It was shown by Abello et al. that through ear decomposition, it suffices to look at staircase polygons.



Terminology

- We consider a simple non-degenerate collection of points in the plane that produces a polygon.
- It was shown by Abello et al. that through ear decomposition, it suffices to look at staircase polygons.
- Two vertices of the polygon are called internally visible if the closed line segment between them is either an edge of the polygon or lies entirely in the interior of the polygon (Abello et al).
- The visibility graph of a polygon is a graph whose vertex set is the same as the vertex set of a polygon and whose edges are the straight-line segments between internally visible vertices.



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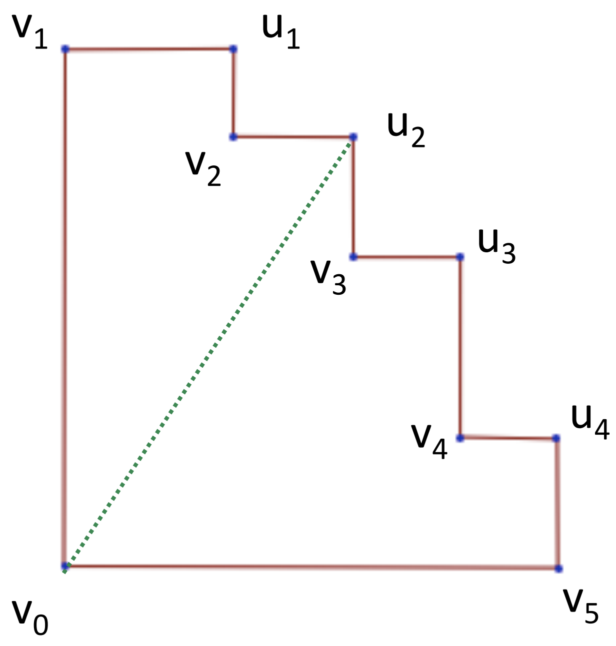

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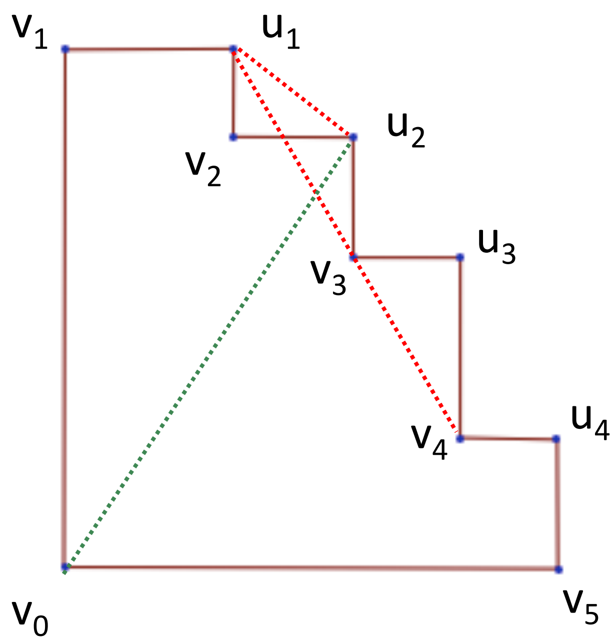


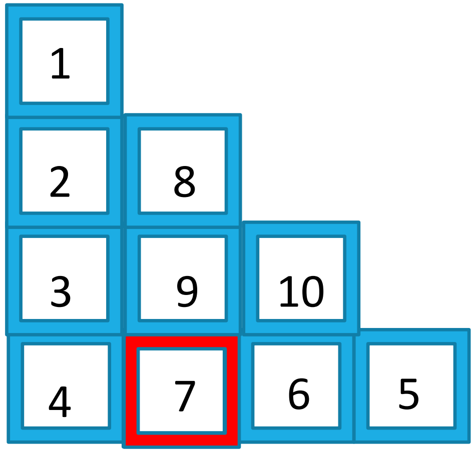
Balanced Tableau

- Staircase shape with n choose 2 consecutive entries

1			
2	8		
3	9	10	
4	7	6	5

Balanced Tableau

- Staircase shape with n choose 2 consecutive entries
- Define the hook of a cell to be the collection of cells that includes the chosen cell with all the cells above it and all the cells to the right



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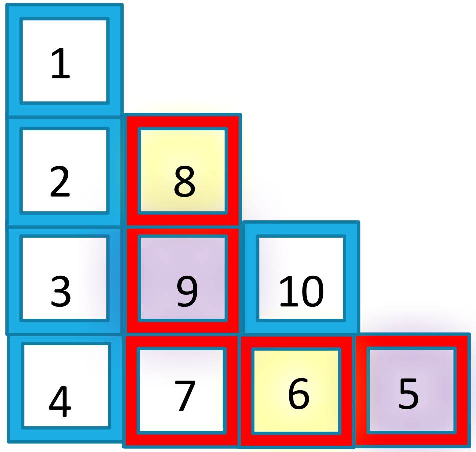
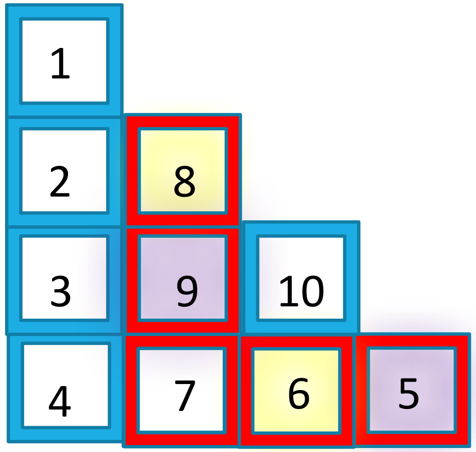
- Define the hook of a cell to be the collection of cells that includes the chosen cell with all the cells above it and all the cells to the right
- Mate cells with respect to 7



Balanced Tableau

- Staircase shape with n choose 2 consecutive entries
- Define the hook of a cell to be the collection of cells that includes the chosen cell with all the cells above it and all the cells to the right
- Mate cells with respect to 7
- A tableau is balanced if the value of every cell lies in between every pair of mate cells in its hook



Local Maxima Rule

- Obtain the Local Maxima (Minima) Tableau from a Balanced Tableau
- It is a collection of subdiagonal entries of an adjacency matrix of a graph

1			
2	8		
3	9	10	
4	7	6	5

1			
1	1		
1	1	1	
1	0	0	1

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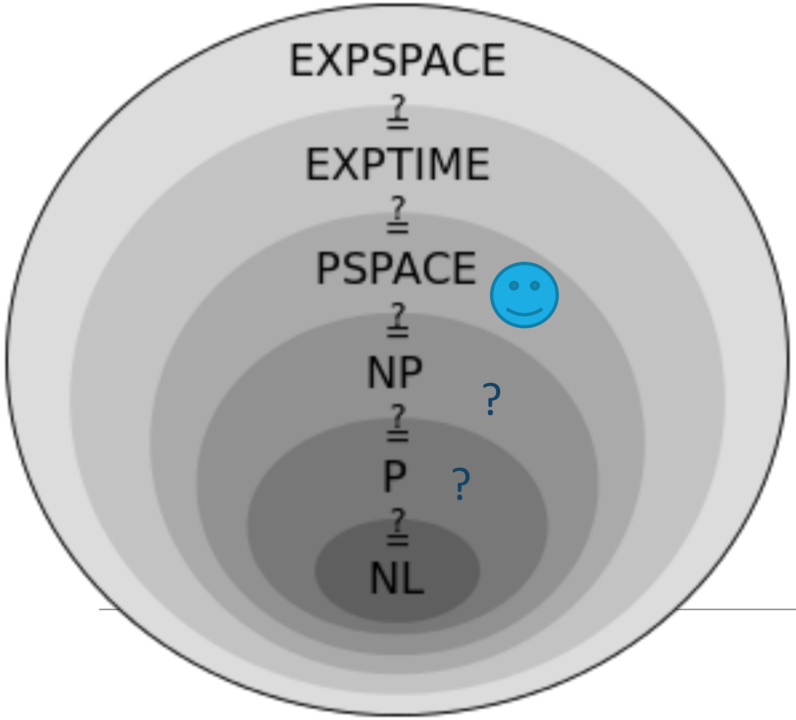
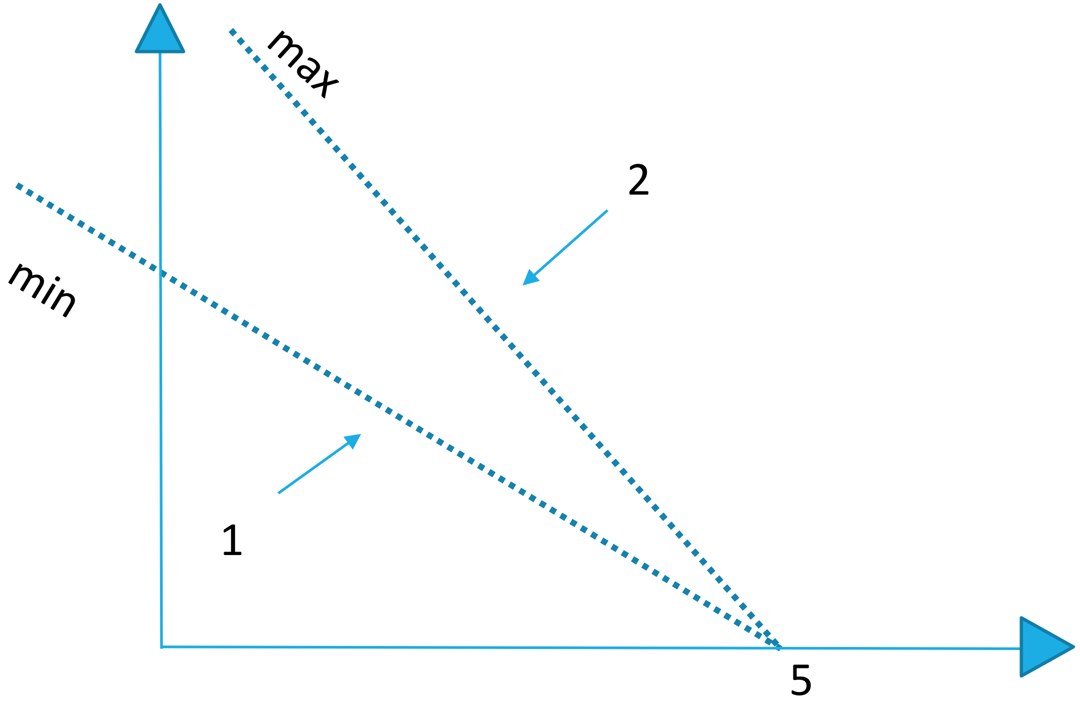


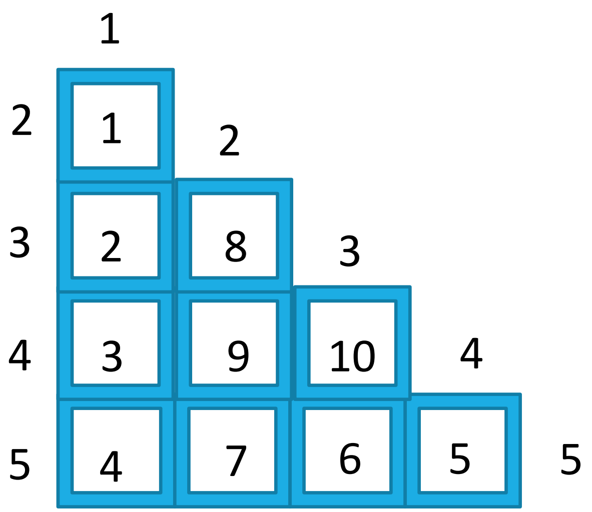
Problem Statement:
Input: A balanced tableau T_n
Output: Build a staircase polygon s.t. its visibility graph is isomorphic to localMax (T_n)

- The problem is known to be PSPACE
- We also want to know whether it is NP or P

Approaches:

- Visibility Cones (Using Combinatorial Convex Hull)





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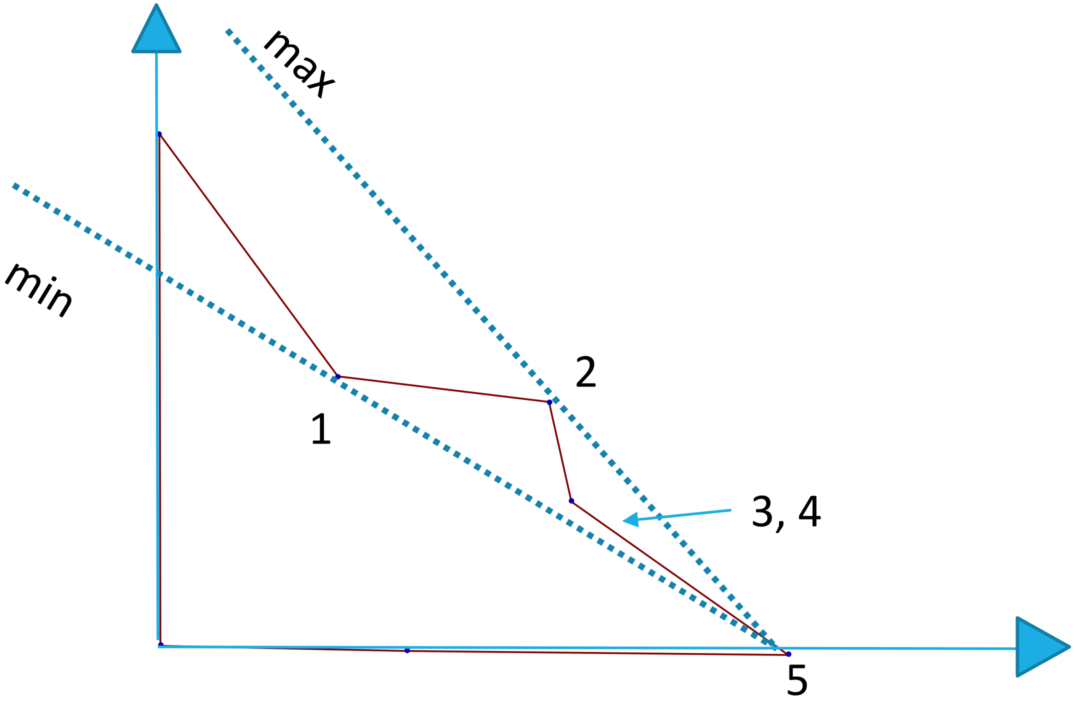
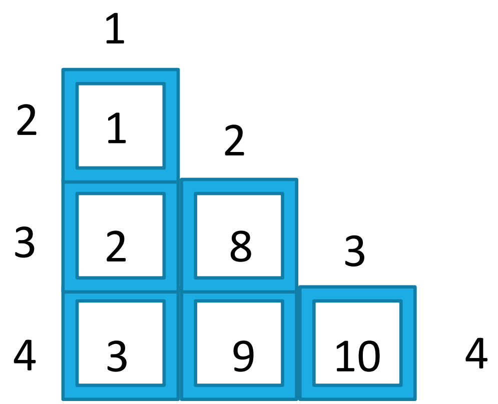


	1	2	3	4
2	1			
3	2	8		
4	3	9	10	
5	4	7	6	5

Approaches:

- Top Down Extension:

    - Build a visibility graph for the balanced tableau without the last row

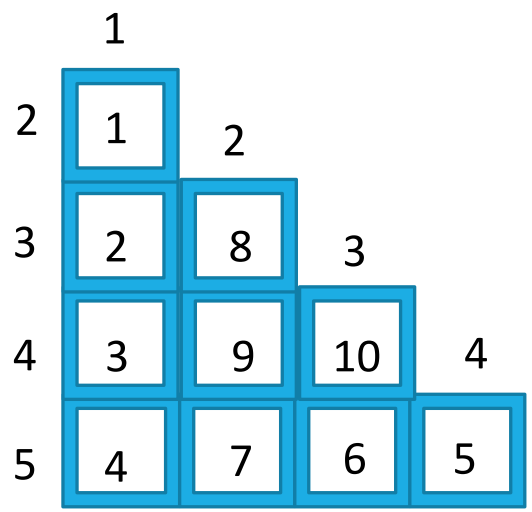


Approaches:

- Top Down Extension:

  - Build a visibility graph for the balanced tableau without the last row

  - Inductively construct the graph with the last row



Thank you!

References:

- [1] Abello et al, *Visibility Graphs of Staircase Polygons and the Weak Bruhat Order, I: from Visibility Graphs to Maximal Chains**. Discrete & Computational Geometry. 1995. 331-358.