Truth Learning in a Social Network

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- Say there is an underlying ground truth that a network seeks to learn (e.x. whether it will rain tomorrow). As the network grows, a greater fraction of the population should ideally learn the ground truth
- However, the network may not learn the ground truth even if everyone is perfectly rational and the majority of people initially believe in the ground truth

We have *n* perfectly rational weather forecasters, each of whom has made some private measurements with independent accuracy q = 2/3 to assess whether it will rain tomorrow.

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Asymptotic truth learning: with probability 1 - o(1), n - o(n) agents correctly learn the ground truth.

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Main question: what are necessary/sufficient conditions for a social network to support asymptotic truth learning?

Graph Topologies: How can we design graphs that aggregate/diffuse info properly?



Image: A matrix

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Are there networks that achieve asymptotic learning for some values of q but not others?

• In the case of a tie, does the tie-breaking strategy affect asymptotic learning?

Robustness: what networks are robust to random / strategically chosen adversaries that disseminate false information?



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