

DIMACS: Schubert Calculus

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Project

- Proving an identity about curve neighborhoods in the Grassmannian

Grassmannian

- $Gr(m,n)$ is the set of all m -dimensional subspaces in a vector space of dimension n
- $GL(n)$ is the set of all invertible $n \times n$ matrices
- Borel subgroup is a subgroup of $GL(n)$ which is the set of all upper triangular matrices
- Maximal Tori is a subgroup of the Borel subgroup which is the set of all diagonal matrices

Schubert Calculus

- Schubert Calculus is the manipulation of Schubert cells
- Schubert cells are orbits of the action of the Borel subgroup on $Gr(m,n)$
- Schubert variety is the closure of the Schubert cells

Curve Neighborhoods

- Let $X = \text{Gr}(m,n)$
- $\Omega \leq X$, closed subset
- $\Gamma_d(\Omega)$ – closure of the union of all curves of degree d in X that intersect Ω

Curve Neighborhoods (cont.)

- Take the curve neighborhood of a Schubert variety to obtain a new Schubert variety of higher dimension
- How are these two varieties related?