Investigating the Vietoris-Rips Complex

Searching for links in data sets
Simplicial Complexes

A Geometric Approach

- A set of Interconnected Simplices:
  - Points
  - Lines
  - Triangles
  - Tetrahedron
  - Pentachoron
  - Etc.
Simplicial Complexes (continued)

A Combinatorial Approach

Let; $A = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, ..., \vec{v}_{n+1}\} \subset \mathbb{R}^{n+1}$; a basis of $\mathbb{R}^{n+1}$.

Let $S_A$ be the smallest convex set s.t. $A \subset S_A \subset \mathbb{R}^{n+1}$.

We call this new set an $n$-simplex
Let $X \subset \mathbb{R}^n$ be a data set:

$$X = \{x_1, x_2, x_3, \ldots, x_m\}$$

Define $2^X$ to be the set of all subsets of $X$.

We call $\bar{X}$ a simplicial complex on $X$ if $\bar{X} \subset 2^X$.

(A subset of the set of all subsets of $X$.)

Note: our simplicial complex must also fulfill the consistency condition:
If $A$ is an element of a simplicial complex then every subset of $A$ is also in the simplicial complex. (this makes sure that we include the boundaries of every simplex)
Consider a specific Simplicial complex \( \tilde{X}_\epsilon \).

This set is defined by: for all \( Y \in \tilde{X}_\epsilon \), for all \( x_i, x_k \in Y \), we have that \( d(x_i, x_k) < \epsilon \).

(we have bounded the pairwise length between any two points in the simplices that make up the simplicial complex)

We call \( \tilde{X}_\epsilon \) the Vietoris-Rips Complex for the specific radius \( \epsilon \).
Constructing the Vietoris-Rips Complex

- Consider the following set of points: $A = \{(0,1), (1,0), (0,-1), (-1,0)\}$
Displayed Simplices:
[  
  \{(0,1)\}; \{(1,0)\}; \{(0,-1)\}; \{(-1,0)\};
\{(0,1),(1,0)\}; \{(1,0),(0,-1)\}; \{(0,-1),(-1,0)\}; \{(-1,0),(0,1)\};
\{(0,1),(1,0),(0,-1)\}; \{(1,0),(0,-1),(-1,0)\}; \{(0,-1),(-1,0),(0,1)\}; \{(-1,0),(0,1),(1,0)\};
\{(0,1),(1,0),(0,-1),(-1,0)\}  ]
A Better Picture:
Cycles

• What we are looking for are cycles.
• Can be thought of as holes in the Vietoris-Rips Complex.
• We are interested in these holes because they are topologically distinct from just points.

• More specifically, we are searching for methods of detecting links; non-intersecting, but interlocked cycles.

• Borromean Rings:
Using Perseus

• One method being investigated is using the program “Perseus”
  – Perseus is a program that accepts a data set and returns descriptions of the generated cycles and overall number of simplexes as the value of epsilon is increased for the given Vietoris-Rips Complex.
  – Possible to track the life of different cycles until they are swallowed by increasingly loose distance requirements. (Increasing value of epsilon)

• Discretization of Convenient Functions
  – We can take already conveniently constructed functions displaying certain qualities (like a link) and choose a discrete subset of the functions to test if Perseus can determine the behavior of the “data”, or if a tweaked code can even detect the links.

• Possibility of applying to real life data to determine behavior.
References:

- Vietoris–Rips Complexes of Planar Point Sets
  Erin W. Chambers, Vin de Silva, Jeff Erickson and Robert Ghrist
  Discrete & Computational Geometry, 2010, Volume 44, Number 1,
  Pages 75-90

- Borromean Rings: Mohamed Ibrahim [www.clker.com](http://www.clker.com)
- Perseus Program: [Vidit Nanda](http://ViditNanda)
- Robet Ghrist