

Fine-grained Space Complexity

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This research is part of a project that has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 823748.



Fine-grained complexity

EDIT DISTANCE

3-SUM

ORTHOGONAL VECTORS

ALL-PAIRS SHORTEST-PATHS

Fine-grained complexity

EDIT DISTANCE

time

$\mathcal{O}(n^2)$

3-SUM

$\mathcal{O}(n^2)$

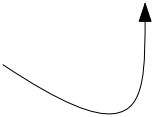
ORTHOGONAL VECTORS

$\mathcal{O}(n^2)$

ALL-PAIRS SHORTEST-PATHS

$\mathcal{O}(n^3)$

basically brute force



Fine-grained complexity

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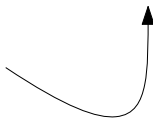
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Can we do better?

basically brute force



Fine-grained complexity

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$\mathcal{O}(n^c / \text{polylog}(n))$
sometimes

basically brute force

Fine-grained complexity

EDIT DISTANCE

3-SUM

ORTHO

ALL-PA

$\mathcal{O}(n^{c-\epsilon})????$

time

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Fine-grained complexity

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If *Strong Exponential Time Hypothesis* holds, then no.

time

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Can we do better?

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Fine-grained complexity

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“If brute force is optimal of NP, then brute force is optimal for some problems in P.”

– not Tung

Fine-grained complexity

space?

~~time~~

Can we do better?

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3-SUM

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Fine-grained complexity

?

~~time~~ **space?**

EDIT DISTANCE $O(n^2)$

3-SUM $O(n^2)$

ORTHO $O(n^c / \text{polylog}(n))$
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ALL-PA $O(n^{c-\epsilon})$????

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Can we do better?

The diagram illustrates the relationship between time and space complexity for various problems. A red arrow points from a question mark to the word 'space?'. The word 'time' is crossed out with a red line. A box containing the question 'Can we do better?' has an arrow pointing to the complexity class $O(n^c / \text{polylog}(n))$ for the ORTHO problem. Another arrow points from the $O(n^2)$ complexity class for EDIT DISTANCE and 3-SUM to the $O(n^c / \text{polylog}(n))$ class. A large box contains a quote about the optimality of brute force in NP and a note that it is not Tung.

Fine-grained complexity

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 $O(n^2)$
 $O(n^2)$

Can we do better?

$O(n^c / \text{polylog}(n))$
sometimes