# Burning Number

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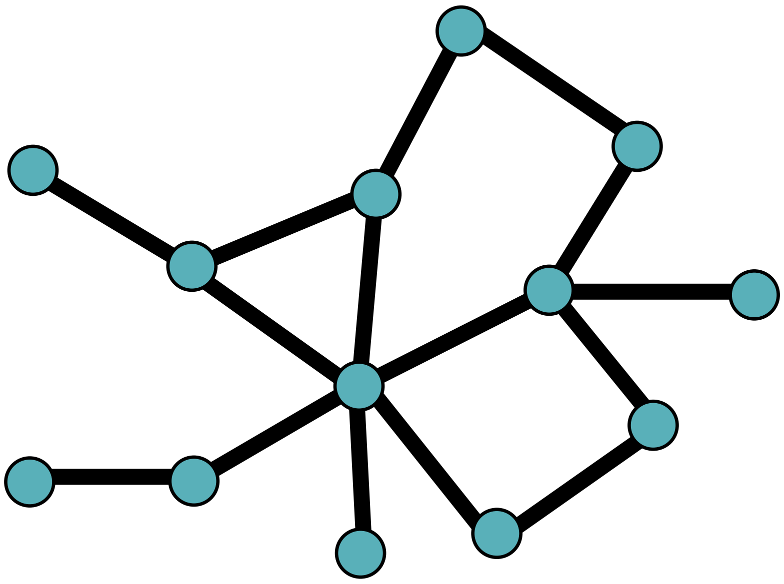
# How to burn a graph

**Burning process**

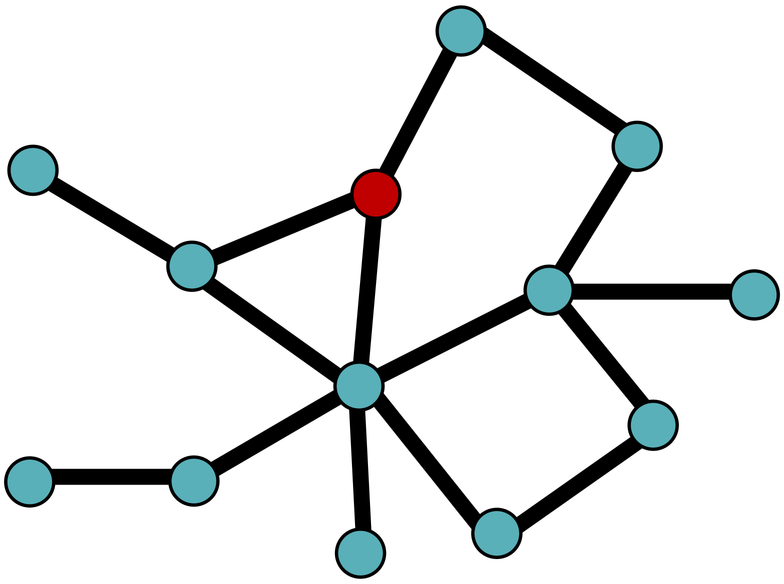
1) All vertices are unburnt



# How to burn a graph

**Burning process**

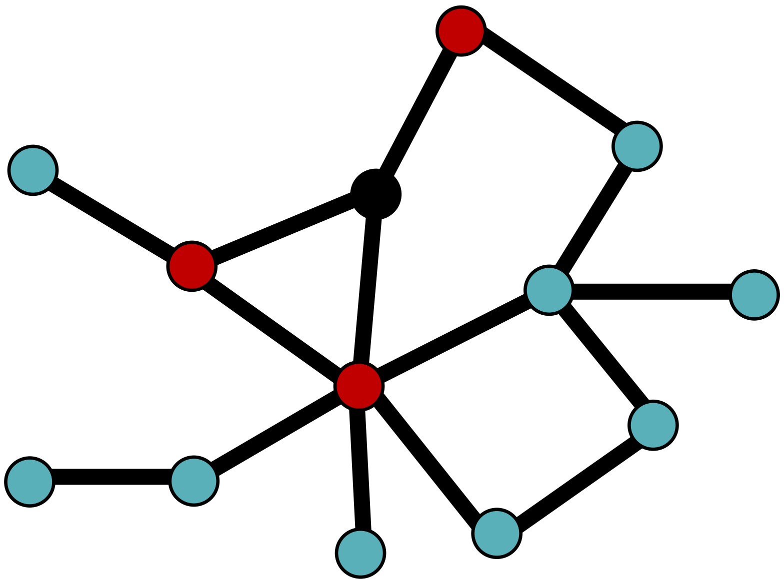
1) All vertices are unburnt
2) Choose a vertex to burn



# How to burn a graph

**Burning process**

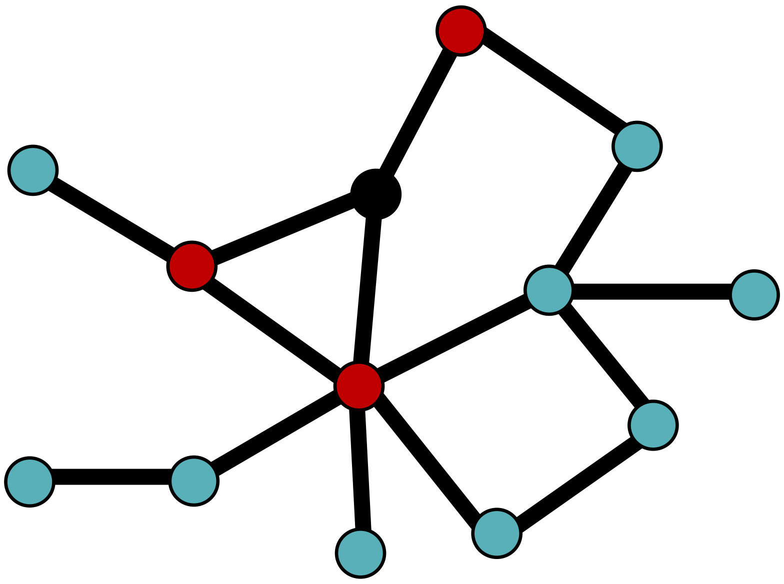
1) All vertices are unburnt
2) Choose a vertex to burn
3) Every burning vertex burns its neighbors



# How to burn a graph

**Burning process**

1) All vertices are unburnt
2) Choose a vertex to burn
3) Every burning vertex burns its neighbors
4) Repeat until all the vertices are burnt



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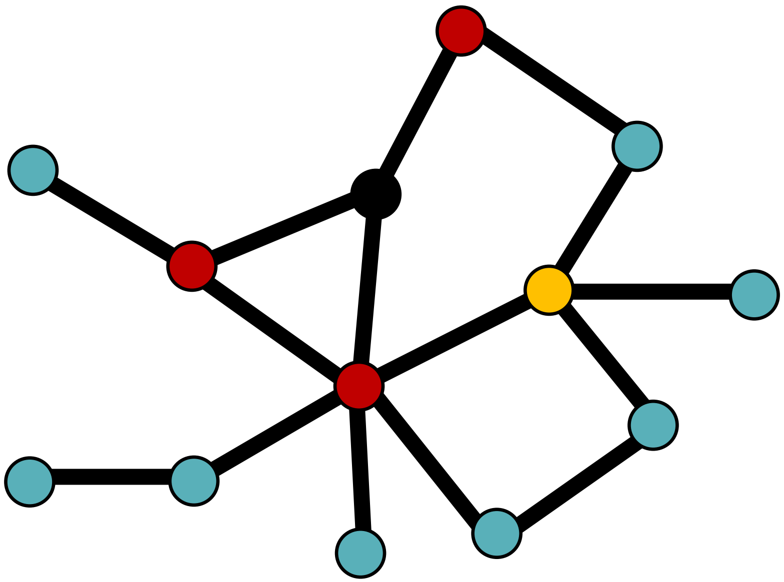

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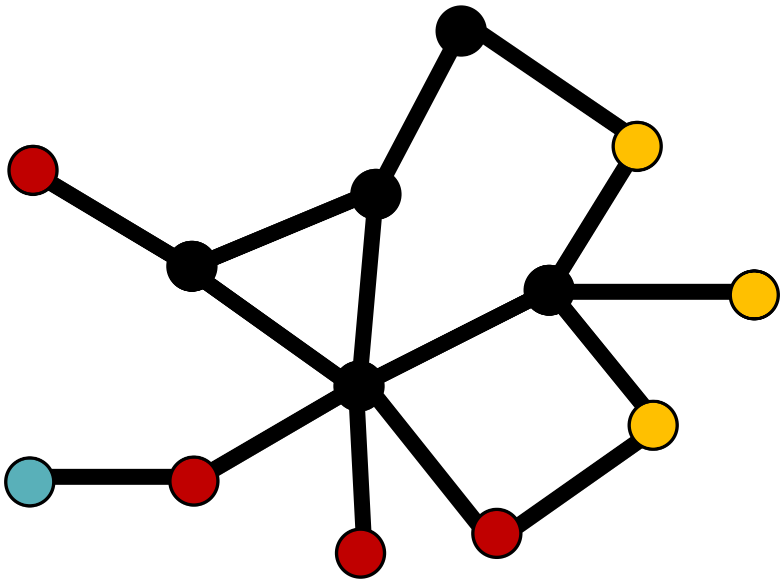

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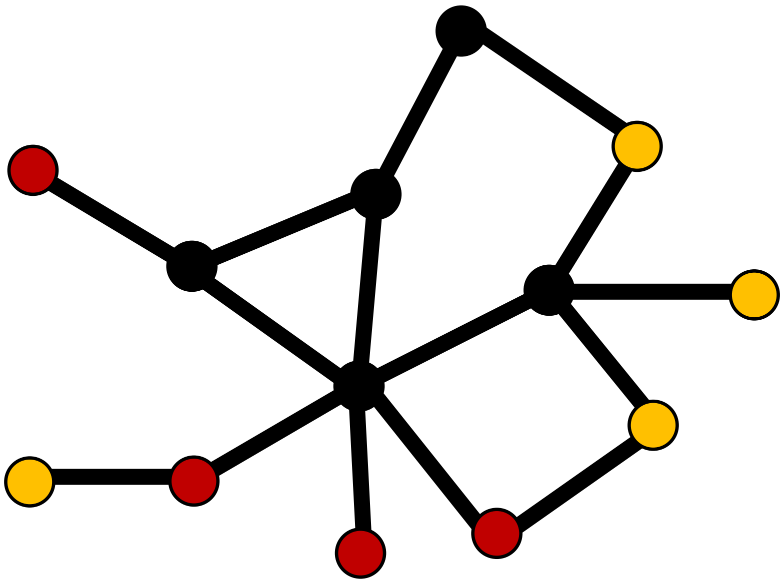

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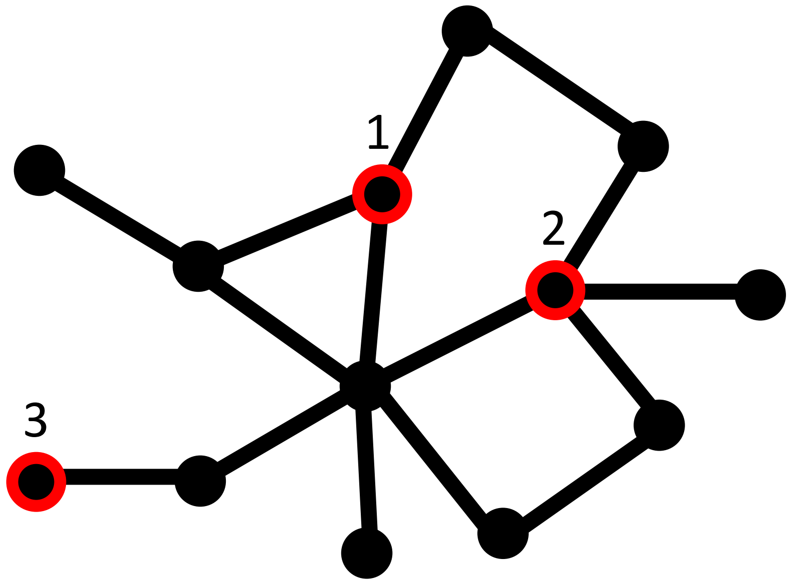


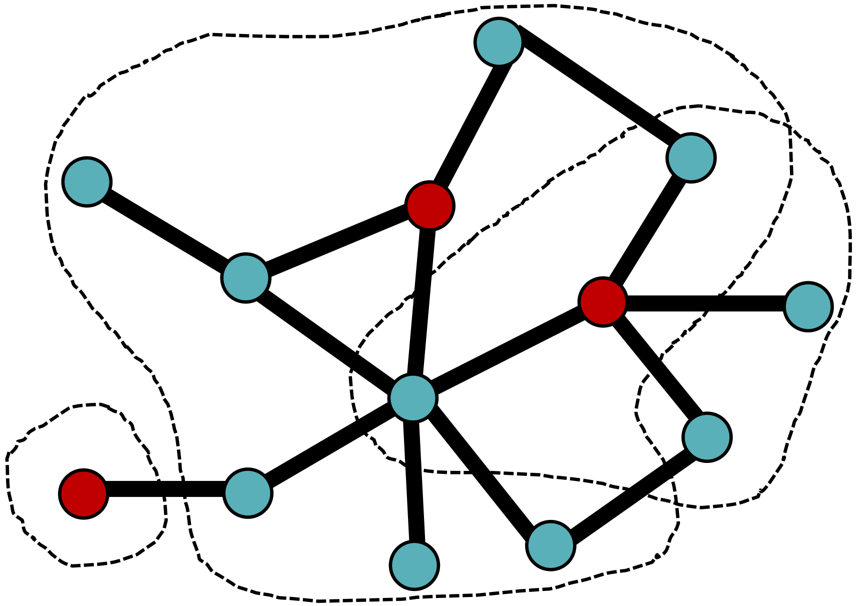

This graph can be burnt in 3 steps
using marked sequence of vertex choices

# Burning number

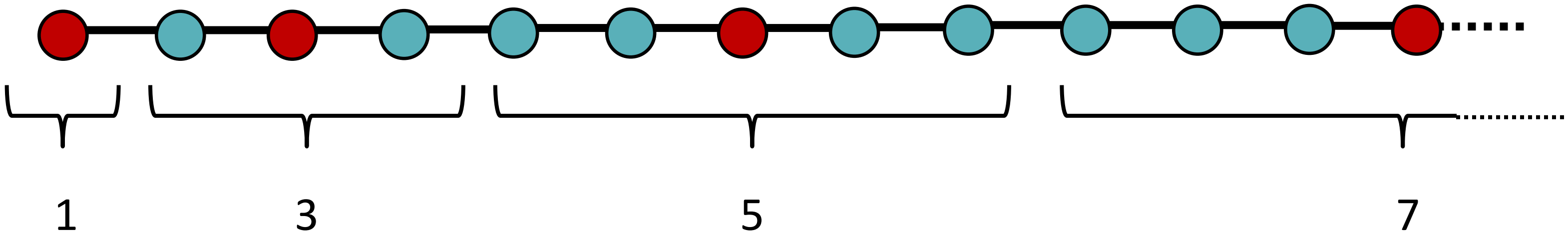
Definition: Burning number of a graph G is the minimum number of burning steps required to burn a graph.

Definition: Burning number of a graph G is the length of the shortest burning sequence.

Definition: Burning number of a graph G is the size of minimum dominating set with increasing radius of dominance



# Burning a path



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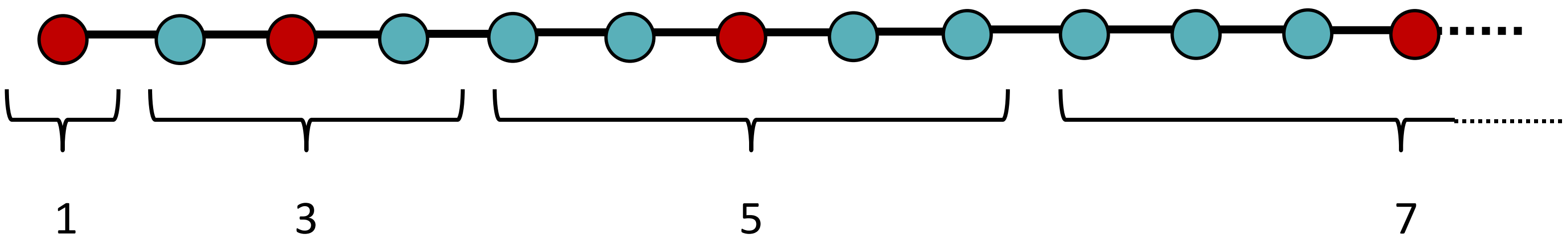
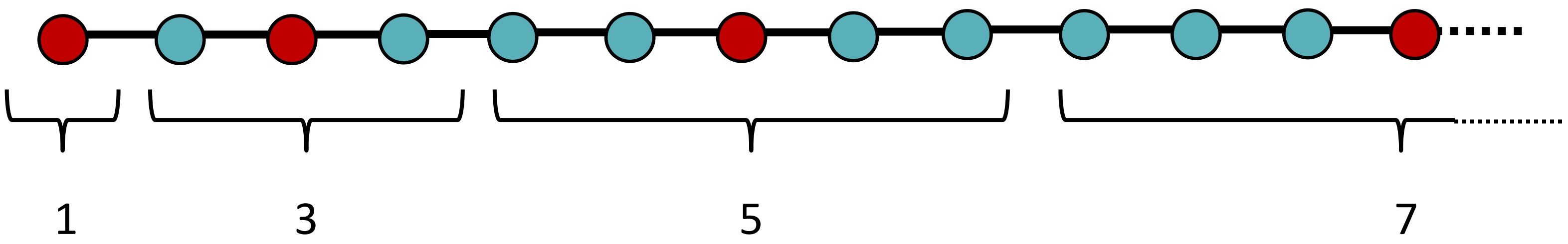



$$\sum_{i=1}^{k}(2i-1)$$

# Burning a path



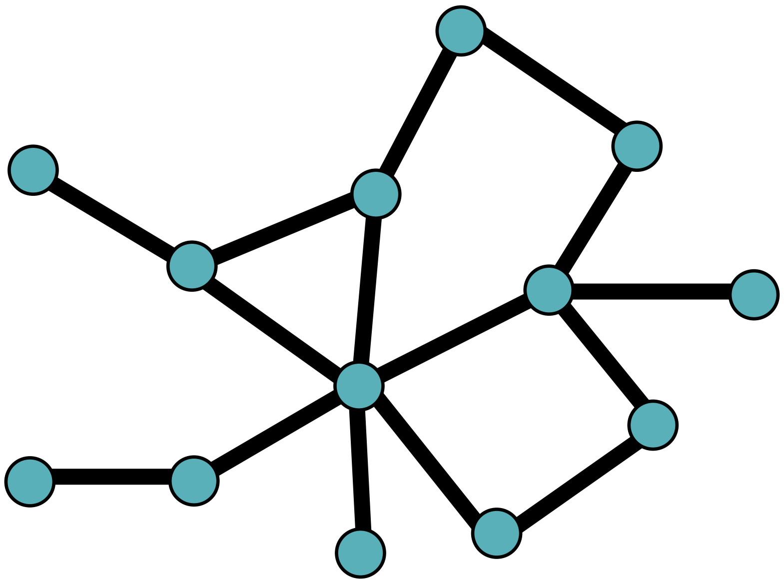

$$\sum_{i=1}^{k}(2i-1)=k^2$$

Observation: Burning number of a path (or cycle) on n vertices is $\lceil\sqrt{n}\,\rceil$.

Hypothesis: Burning number of any graph on n vertices is at most $\lceil\sqrt{n}\,\rceil$.

# Simple upper bound

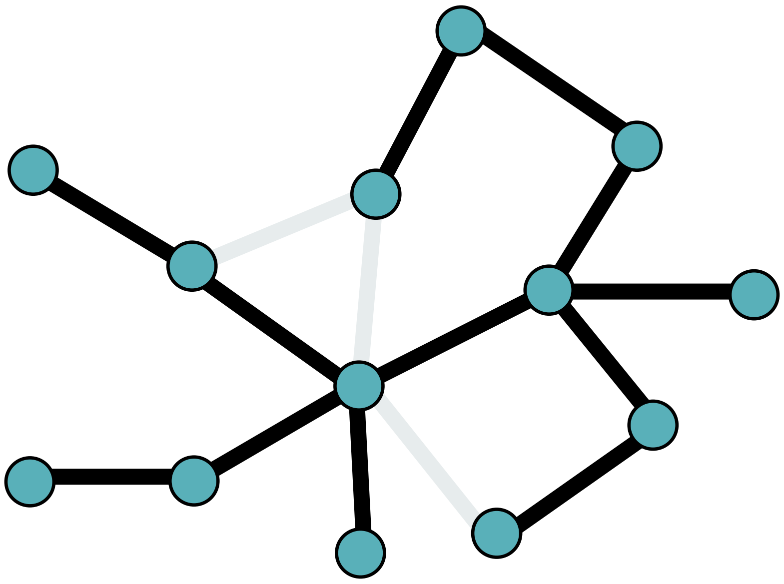
<u>Theorem:</u> Burning number of any graph is at most $\lceil \sqrt{2n} \rceil$.



# Simple upper bound

Theorem: Burning number of any graph is at most $\lceil \sqrt{2n} \rceil$.

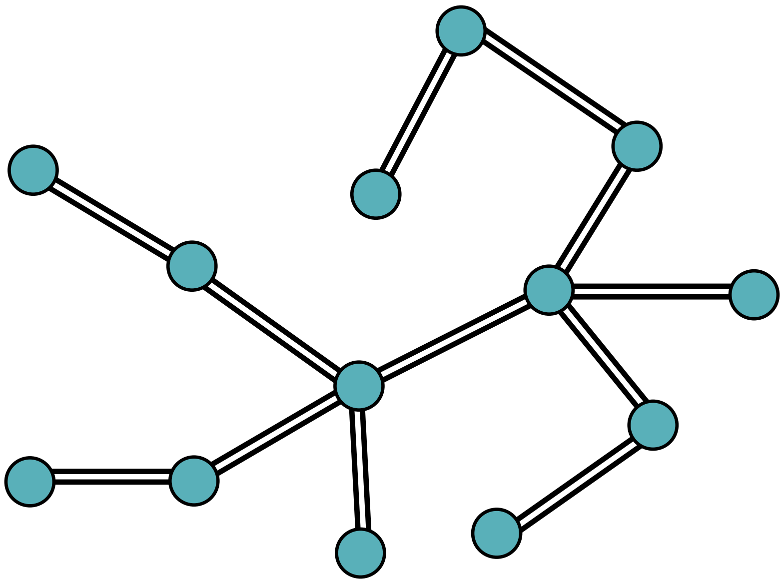
1) Take a spanning tree of a graph



# Simple upper bound

Theorem: Burning number of any graph is at most $\lceil \sqrt{2n} \rceil$.

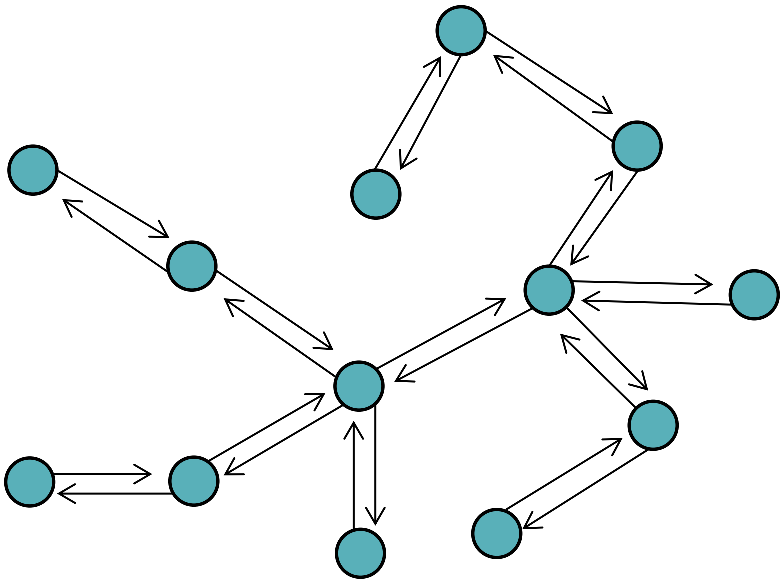
1) Take a spanning tree of a graph
2) Double all edges



# Simple upper bound

Theorem: Burning number of any graph is at most $\lceil \sqrt{2n} \rceil$.

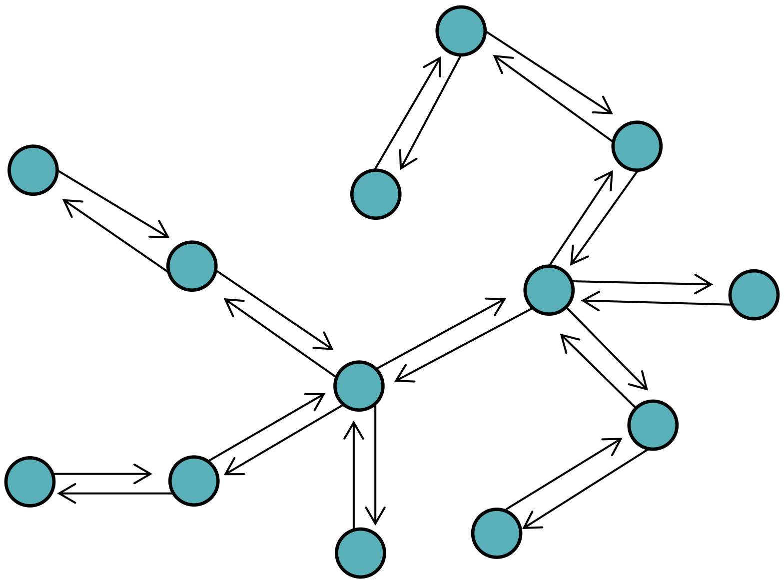
1) Take a spanning tree of a graph
2) Double all edges
3) Find an Eulerian cycle
4) Cycle has 2n-2 vertices



# Simple upper bound

Theorem: Burning number of any graph is at most $\lceil \sqrt{2n} \rceil$.

1) Take a spanning tree of a graph
2) Double all edges
3) Find an Eulerian cycle
4) Cycle has 2n-2 vertices
5) Resulting cycle can now be burned same as on the previous slide using $\lceil \sqrt{2n-2} \rceil$ vertices



# Known upper bounds

$$bn(G) \leq \sqrt{\frac{32}{19}\frac{n}{1-\varepsilon}} + \sqrt{\frac{27}{19\varepsilon}}$$

for any $0 < \varepsilon < 1$

$$bn(G) \leq \lceil \sqrt{n} \rceil + n_{\geq 3}$$

$n_{\geq 3}$ is the number of vertices of degree at least 3

$$bn(G) \leq \left\lceil \sqrt{n + n_2 + \frac{1}{4}} + \frac{1}{2} \right\rceil$$

$n_2$ is the number of vertices of degree 2

# Sources

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## Thank You