Outline

1. Graph Theory Basics
2. Competition Graphs
3. Partial Order
4. Dyck Path
5. Problem
6. Future Work
A graph $G$ is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$ and a relation that associates each edge with two vertices.

A subgraph of $G$ is a graph $H$ where $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$ and the edge vertex assignments are the same.

A directed graph is a graph where each edge has an arrow pointing to a vertex.

The endpoints of a directed edge are called the head and the tail. A directed edge points from the tail to the head.

A graph $H$ is forbidden in $G$ if it is not isomorphic to any subgraph of $G$. 
Examples

**Figure**: A simple graph on 5 vertices

**Figure**: A simple digraph on 5 vertices
A competition graph $C(D)$ is constructed from a digraph $D$ having the same vertex set as $D$ and an edge between two vertices $x$ and $y$ exist if there exists a vertex $w$ such that $(w, x), (w, y) \in A(D)$.

A graph is an interval graph if there exists intervals on the real line such that two vertices are connected by an edge if and only if their corresponding intervals overlap.

The boxicity of $G$ is the smallest $p$ such that we can assign to each vertex of $G$ a box in Euclidean $p$-space so that two vertices are neighbors if and only if their boxes overlap.
Consider an ecosystem where each vertex is a species and an edge from \( x \) to \( y \) exists if \( x \) preys on \( y \).
Figure: Above: The competition graph derived from a simple food web; Below: Shows that this competition graph is an interval graph
A **partial order** is a binary relation over a set $S$ which is reflexive, antisymmetric and transitive. A **partially ordered set**, or poset, is a set with a partial order.

**Figure**: A Hasse diagram on a set of size 3.
A relation $\preceq$ on a subset $S$ of $\mathbb{R}^2$ is a **doubly partial order on $S$** if $(x, y) \preceq (z, w)$ whenever $x < z$ and $y < w$ for $(x, y), (z, w) \in S$.

A double partial ordered set is a set with a doubly partial order. A digraph $D$ is called a **doubly partial order** if $D$ is isomorphic to a digraph of a doubly partial order relation $\preceq$ on a subset of $\mathbb{R}^2$.

**Figure**: A doubly partial order $D$ and a doubly partial order on a subset of $\mathbb{R}^2$ which is isomorphic to $D$
Definition (Choi, Kim, Kim, Lee 2014)

A relation $\prec$ on a subset $S$ of $\mathbb{R}^n$ is an n-tuply partial order on $S$ if, for $(x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots, y_n) \in S$,

$(x_1, x_2, \ldots, x_n) \prec (y_1, y_2, \ldots, y_n)$ whenever $x_i < y_i$ for $i = 1, \ldots, n$.

For an integer $n \geq 2$, a digraph $D$ is called an n-tuply partial order if $D$ is isomorphic to a digraph of an n-tuply partial order relation $\prec$ on a subset of $\mathbb{R}^n$. A graph $G$ is the competition graph of an n-tuply partial order if there is an n-tuply partial order $D$ such that $G$ is isomorphic to the competition graph of $D$. 
A Dyck path is a staircase walk from (0,0) to (n,n).
*Theorem (Cho, Kim 2005)*

*Competition graphs of a digraph of doubly partial order are interval graphs.*

*Theorem (Cho, Kim 2005)*

*An interval graph with sufficiently many isolated vertices is the competition graph of a doubly partial order.*
Previous Results

Theorem (Choi, Kim, Kim, Lee 2014)

Every interval graph can be made into the competition graph of a triply partial order by adding sufficiently many isolated vertices.

Theorem (Choi, Kim, Kim, Lee 2014)

Every tree can be made into the competition graph of a triply partial order by adding sufficiently many isolated vertices.

Corollary (Choi, Kim, Kim, Lee 2014)

If a graph $G$ is the competition graph of a triply partial order, then $G$ does not contain $K_{3,3}$ as an induced subgraph.
Let $D$ be a digraph of n-tuply partial order. What are forbidden subgraphs, if any, of the competition graph $C(D)$ of $D$?
Results

Theorem

If loops are understood a digraph $D$ of $n$-tuplely partial order represents a poset.

Theorem

Given a digraph $D$ of $n$-tuplely partial order a forbidden subgraph of $C(D)$ is $C_m$ where $m \geq 4$.

Theorem

Given a digraph $D$ of $n$-tuplely partial order any subgraph with maximum degree $n - 1$ is forbidden in $C(D)$. 
Future Work

- Continue characterizing forbidden subgraphs in competition graphs
- Generalize to competition graphs constructed from digraphs isomorphic to general posets
Thank You