Hardness of Approximating k-clique

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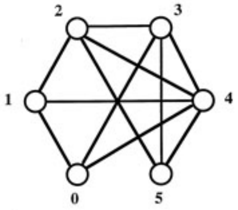
The k-clique Problem

The Problem:

- Input: Graph $G = (V, E)$, integer k
- Output: Find k-clique in G.



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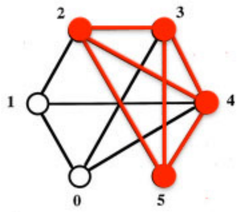


Bad News

- k-clique is NP-complete (i.e., poly(n)-time algorithm unlikely)
- k-clique is W[1]-complete (i.e., $f(k)$poly(n)-time algorithm unlikely)
- no $n^{o(k)}$-time algorithm under ETH
- believed to have no $n^{\omega k/3}$-time algorithm

Approximating k-clique

- trivial: k approximation
- NP-Hard to approximate to $n^{1-\epsilon}$ factor for any $\epsilon > 0$ (Håstad'96)
- W[1]-complete to approximate to constant factor (Lin'21)
- W[1]-complete to approximate to $k^{o(1)}$ factor (Karthik-Khot'21)

Recent Results

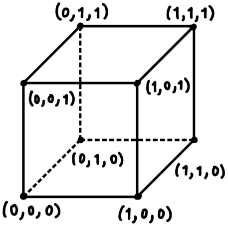
1. Assuming Gap-ETH, no $o(k)$-approx. in FPT time (Chalermsook et al.'17)
2. Assuming ETH, no $o(k/\text{polylog}(k))$-approx. in FPT time (Bafna-Karthik-Minzer'25)
3. Nothing is known in fine-grained world for explicit k.

Motivating Direction

- **current**: use CSPs to study (in)approximability of k-clique
- **want**: obtain results from studying k-clique directly

Current Directions

1. reducing $(i, i+1)$-gap clique to $(i, i+2)$-gap clique for small fixed i
2. using locally-decodable codes to improve self-reductions
3. PCP Theorem proof for cliques via hypercubes



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