LLM-Based Codes for Deletion Channels

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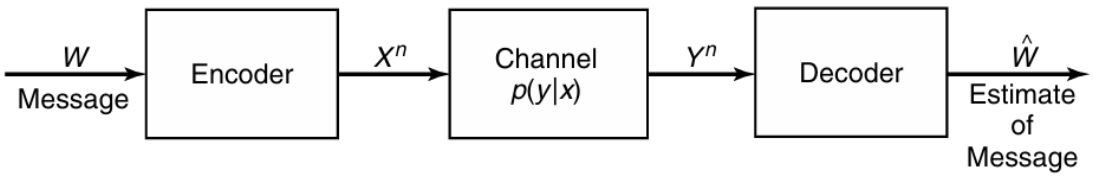
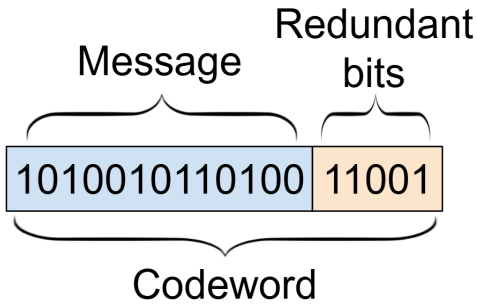
Review of Deletion Channels





$\mathcal{X}$ = channel input alphabet $\quad \mathcal{Y}$ = channel output alphabet

Deletion Channel: Every symbol X_i in message $X \in \mathcal{X}^n$ is dropped from the message with probability p

Example: $\mathcal{X} = \{0, 1\}$, $\mathcal{Y} = \{0, 1\}$

$$X = 1010$$
$$Y = 110$$

Shannon's Noisy-Channel Coding Theorem

Every channel has a capacity $C = \sup_{p(X)} I(X;Y)$.

Theorem (Shannon's Noisy-Channel Coding Theorem)

For any rate $R < C$, there exists a code that achieves R with arbitrarily small probability of error.

Notes:

- Shannon did not tell us *how* to achieve this rate
- The capacity of the deletion channel is not known

Project Vision

Use the English alphabet as input/output alphabet

$\mathcal{X} = \{$A, B, C, ..., X, Y, Z, a, b, c, ..., x, y, z$\}$
$\mathcal{Y} = \{$A, B, C, ..., X, Y, Z, a, b, c, ..., x, y, z$\}$

Exploit inherent redundancy:
"DIACS REUis th coolesresearh pogam ver ceatd!"

Use LLMs as a tool to recover text with deletions

Is GPT Better Than You?

Deleted text ($p = 0.30$):
"heele wee on of three easvewed as bein inthe nnior Rdgers."

Is GPT Better Than You?

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Original:
"The Steelers were one of three teams viewed as being in the running for Rodgers."

Is GPT Better Than You?

Deleted text ($p = 0.30$):
"heele wee on of three easvewed as bein inthe nnior Rdgers."

Original:
"The Steelers were one of three teams viewed as being in the running for Rodgers."

GPT-recovered:
"The Steelers were one of three teams viewed as being in the running for Rodgers."

Creating Datasets

Need for large amounts of plaintext

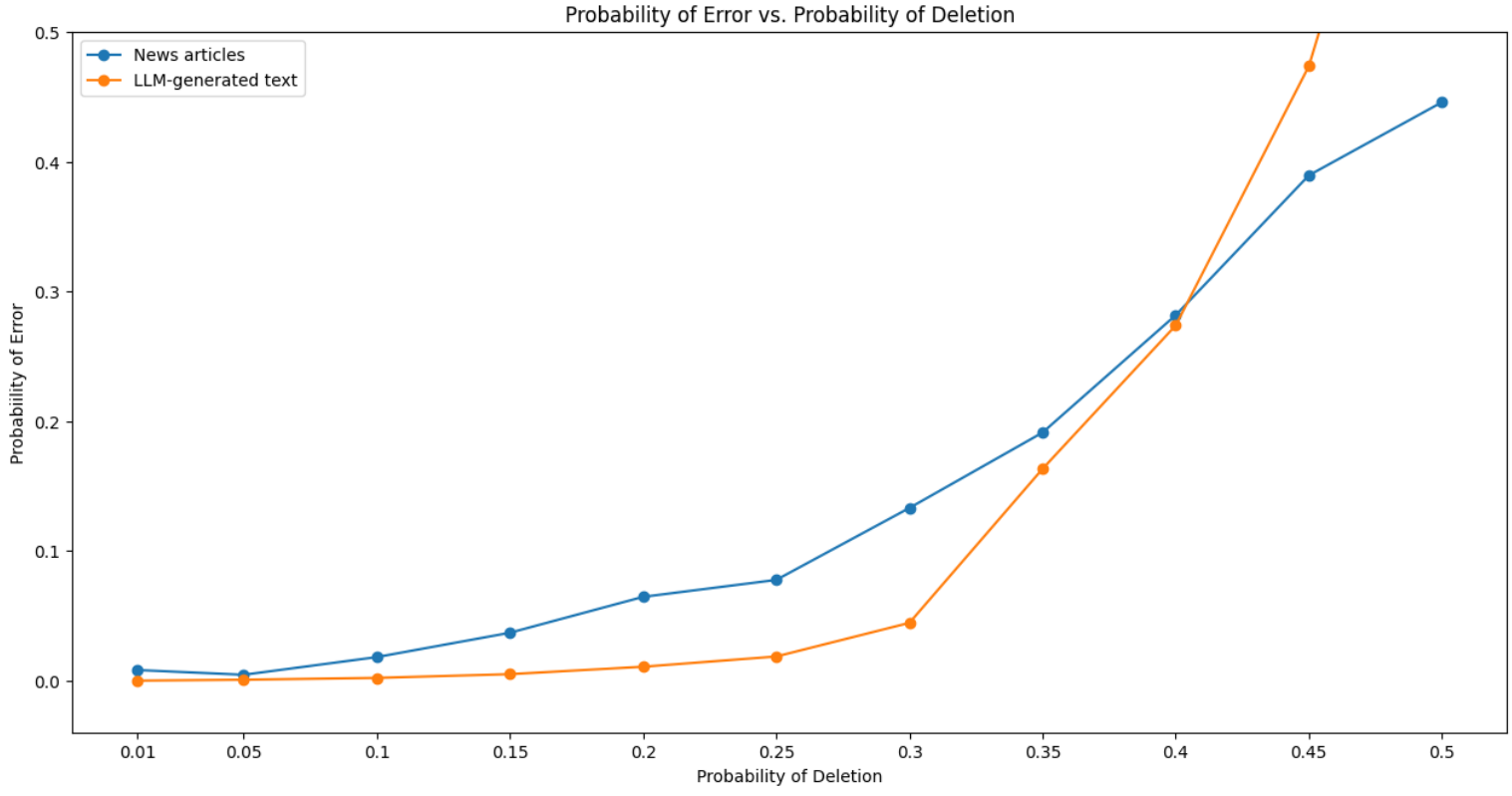
Method 1: Scraping recent news articles

- "After Saturday's 3-1 loss to Turkey..."
- "An international team led by Dr. Andy Rivkin..."
- "What seafood should you avoid?"

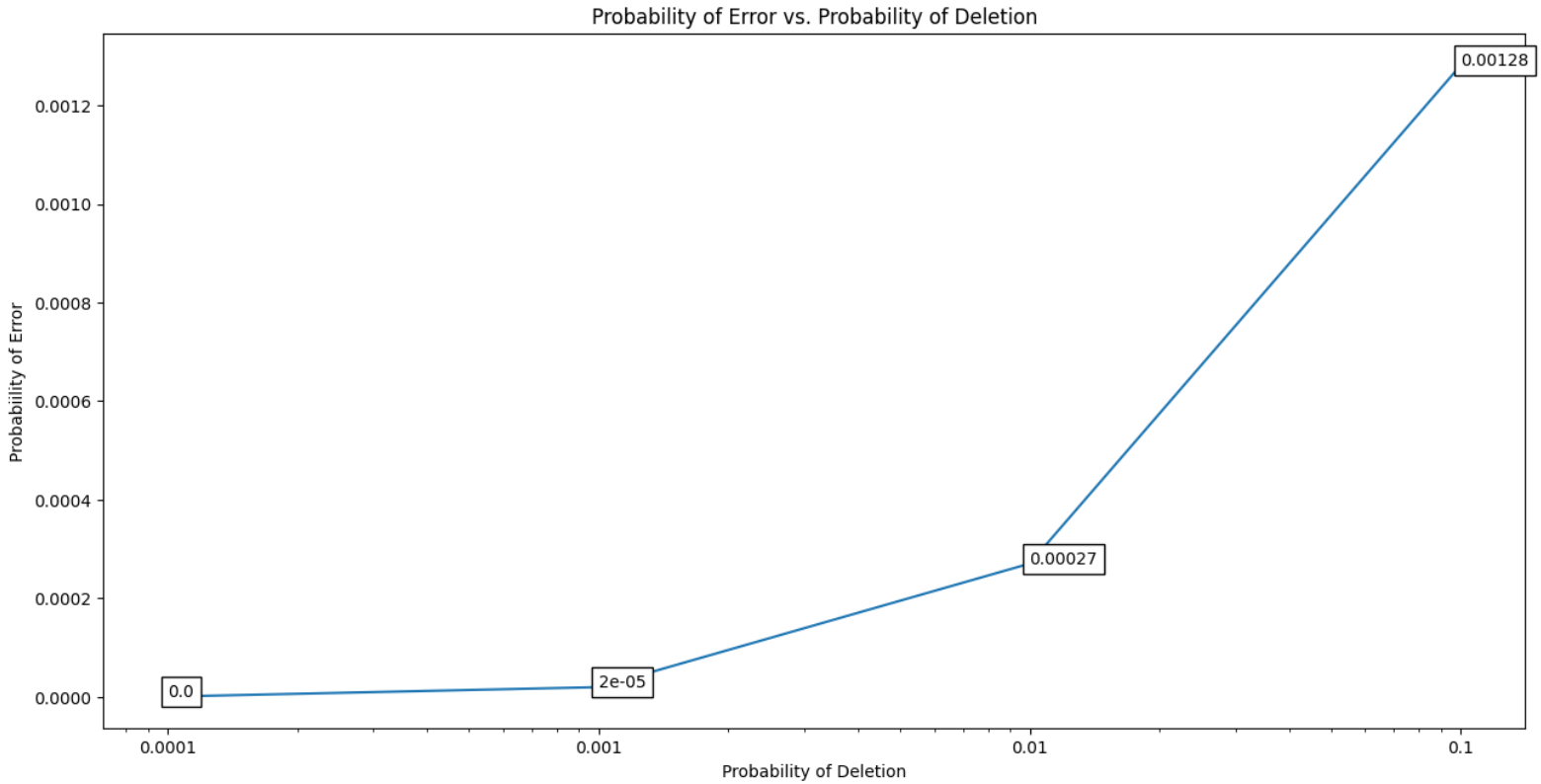
Method 2: Generating "clean" texts using LLMs

- "...divide cashflows into tranches..."
- "...crucial for companies operating loally..."

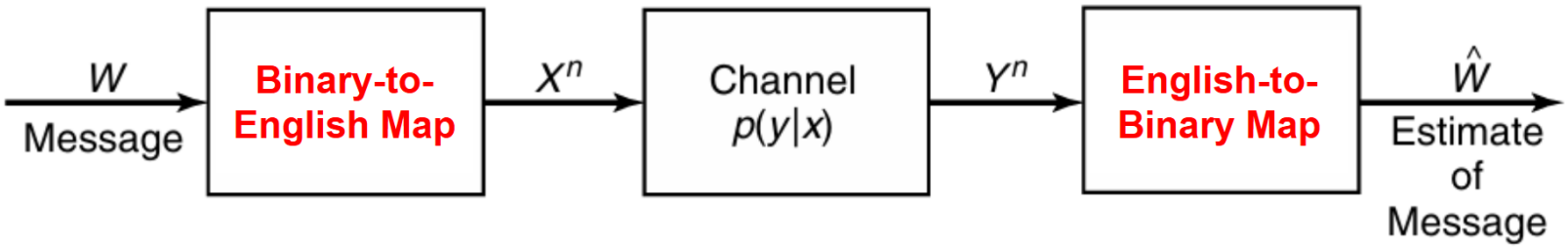
GPT Performance on Datasets



GPT Performance on Datasets



Mapping Binary to a Language



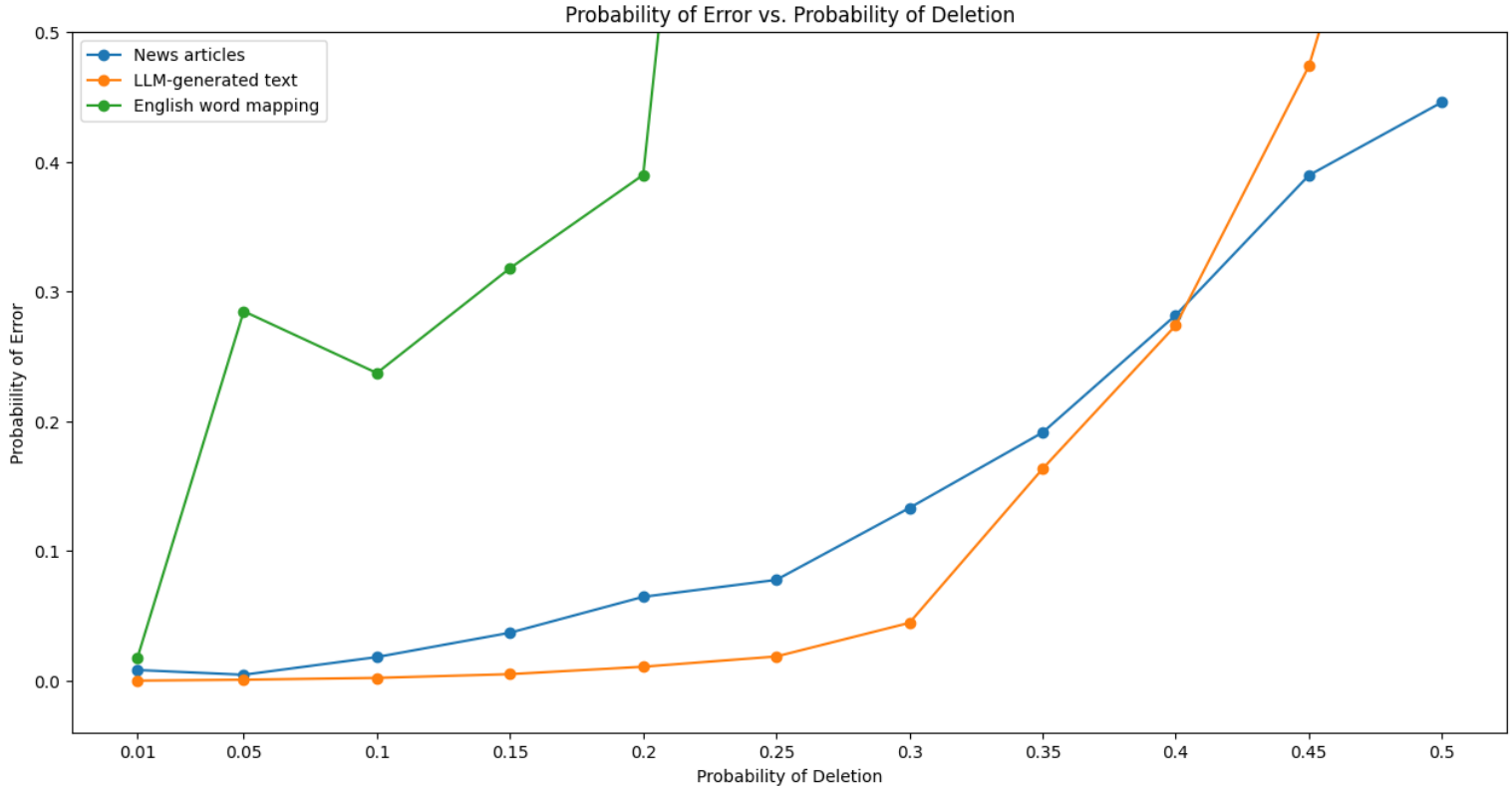
Idea 1: Map all 2^b binary strings of length b to English *words*

- 0000 $\rightarrow$ apple
- 0001 $\rightarrow$ banana
- ...

Idea 2: Map all 2^b binary strings of length b to English *sentences*

- 0000 $\rightarrow$ The sky is blue.
- 0001 $\rightarrow$ P equals NP.
- ...

GPT Performance Using Mapping



Next Steps

- Explore more efficient mappings from binary to some structured language
- Explore alternatives to edit distance (semantic distance)
- Automatic identification of parts of text that cannot be inferred from context
- Theoretical guarantees on performance

Acknowledgments

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