Δ-coloring in the graph streaming model

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- Examples:
  - Median-finding ($O(p)$ pass $\tilde{O}(n^{1/p})$ space algorithms are known).
  - Counting distinct elements ($(1+\epsilon)$-approx in $O(1/\epsilon^2 \cdot \log n)$ space with constant probability of success known).

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- A good survey is [2]

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- For G with maximum degree Δ, $\Delta + 1$ coloring is easy (assign arb. color to some vertex, then color rest "greedily").
- If G is not a clique or an odd-cycle, then it can be Δ-colored (Brook's Theorem).

Our project

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- It is known that randomized $(\Delta + 1)$-coloring is possible in $\tilde{O}(n)$ space [1]
- With randomization, can we Δ-color graphs in $\tilde{O}(n)$ space in $O(1)$ passes?

References

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