BIASED RANDOM WALKS

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Let $G = (V, E)$ be a graph, $a \neq b \in V$. A simple random walk is a randomly generated sequence of vertices $(v_i)$ such that $v_1 = a$, $v_{i+1} \in N(v_i)$ and $v_{i+1}$ is chosen uniformly at random. The hitting time of $b$ is the number of steps the walk needs to reach $b$ from $a$. 
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Let $K_n$ be a complete graph, $n \geq 3$. Then, given two distinct vertices $a, b \in V(K_n)$, the expected hitting time of $b$ is $n - 1$. 
It is known that for every two vertices on any graph, the expected hitting time is not worse than $O(n^3)$.
It is known that for every two vertices on any graph, the expected hitting time is not worse than $O(n^3)$. But what if we help the random walker in some way?
Given a graph $G = (V, E)$, choose some vertices $F \subseteq V$. In these ‘excited’ vertices, the random walker will deterministically take a step along a fixed shortest path. Does the hitting time change, and if so, how?
Returning to the complete graph, if we excite $a$, the expected hitting time becomes 1.
NOT SO FAST

\[ \text{ Trap } \]
Can we show the same $O(n^3)$ bound on the expected hitting time as before?
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Are there any other natural ‘biases’, which help the random walker?
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