Biased random walks

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- Let $G = (V, E)$ be a graph, $a \neq b \in V$. A simple random walk is a randomly generated sequence of vertices (v_i) such that $v_1 = a, v_{i+1} \in N(v_i)$ and v_{i+1} is chosen uniformly at random.

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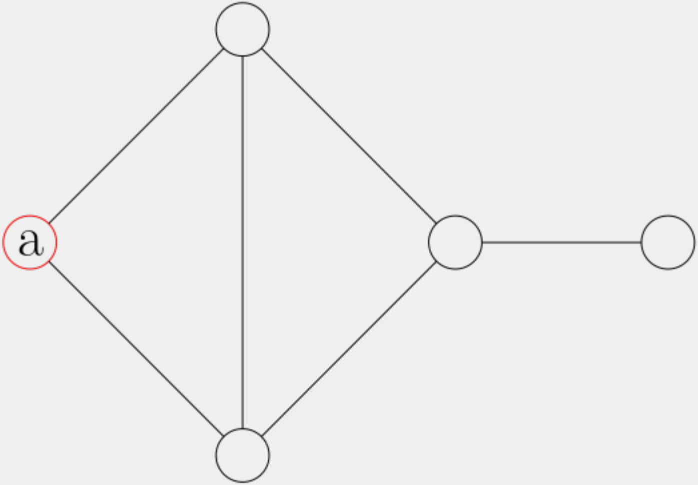
- The hitting time of b is the number of steps the walk needs to reach b from a.

Example



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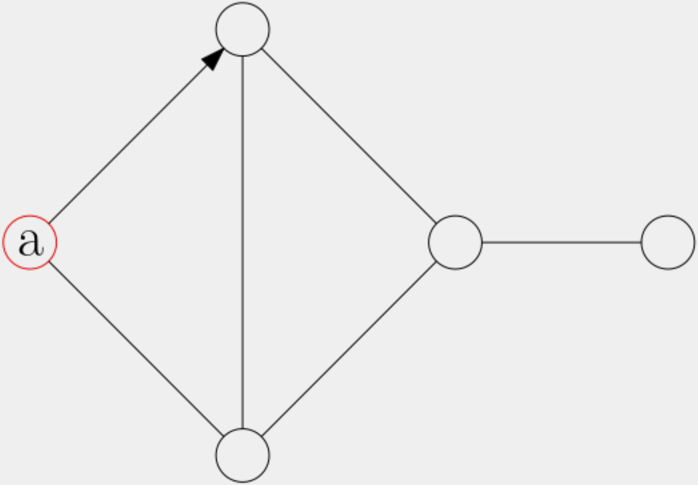

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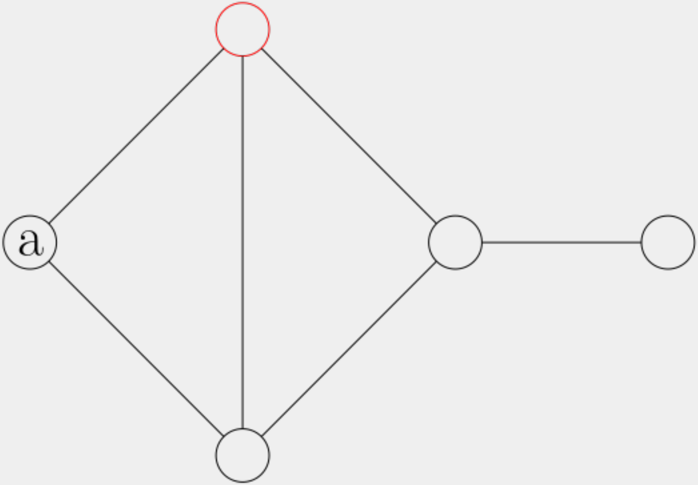

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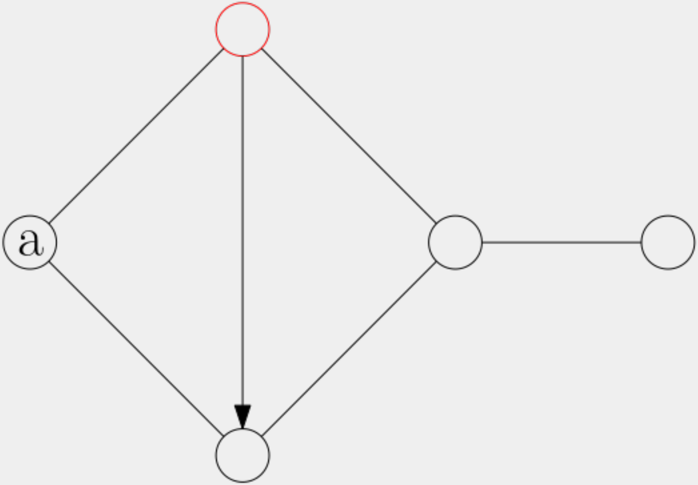

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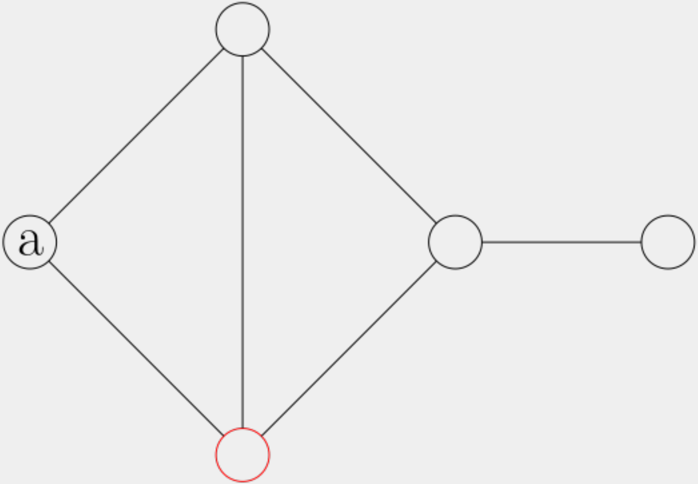

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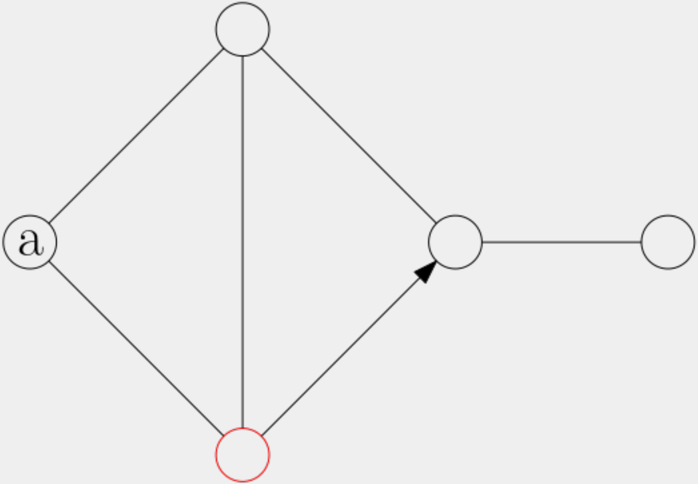

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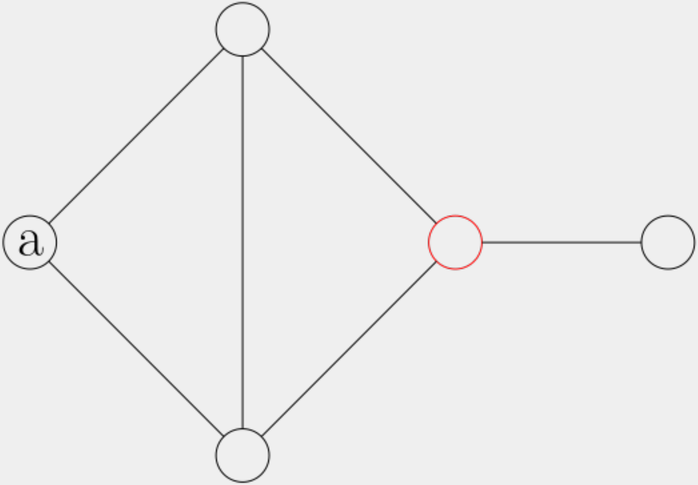

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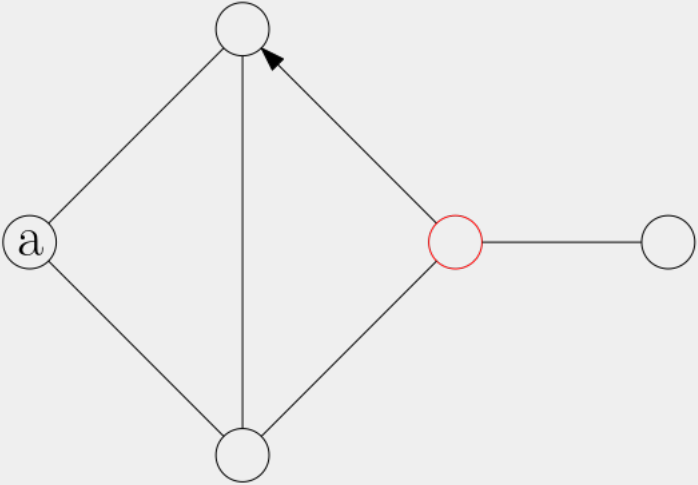

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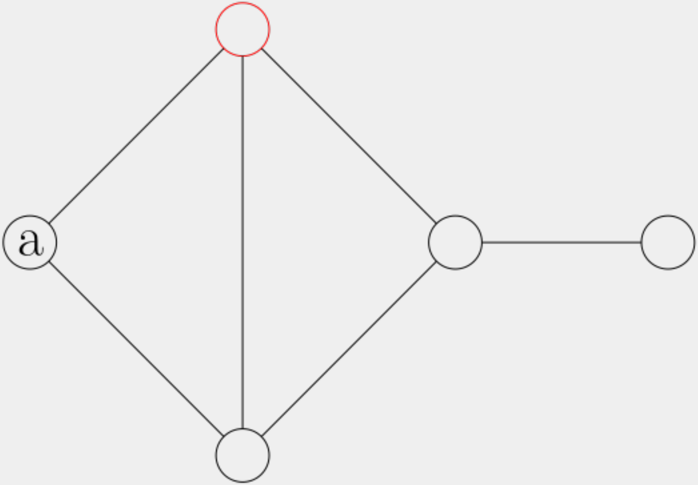

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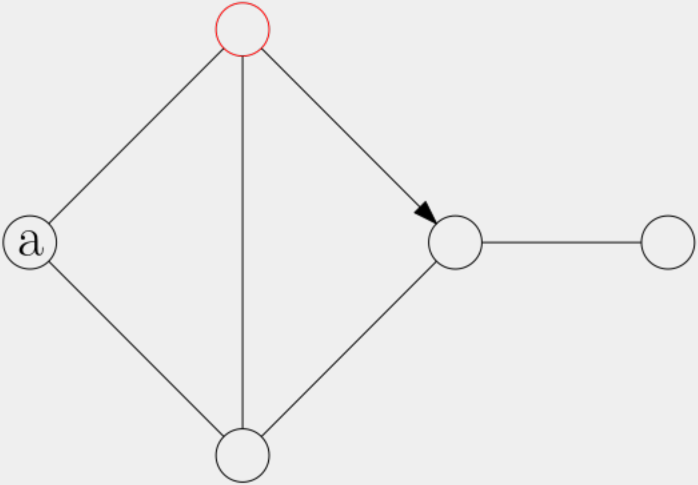

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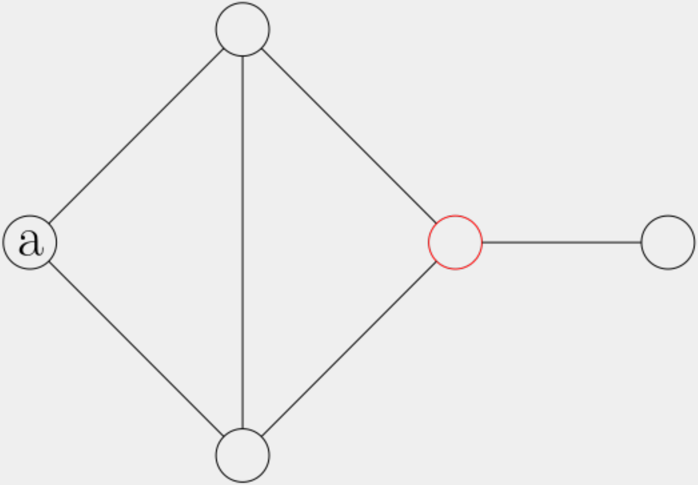

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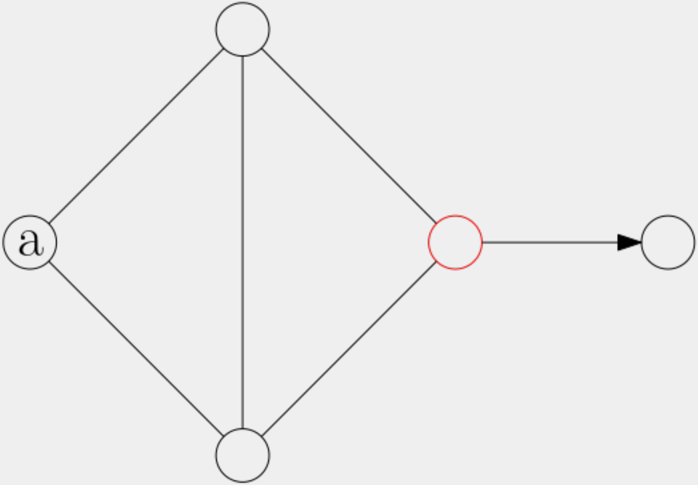

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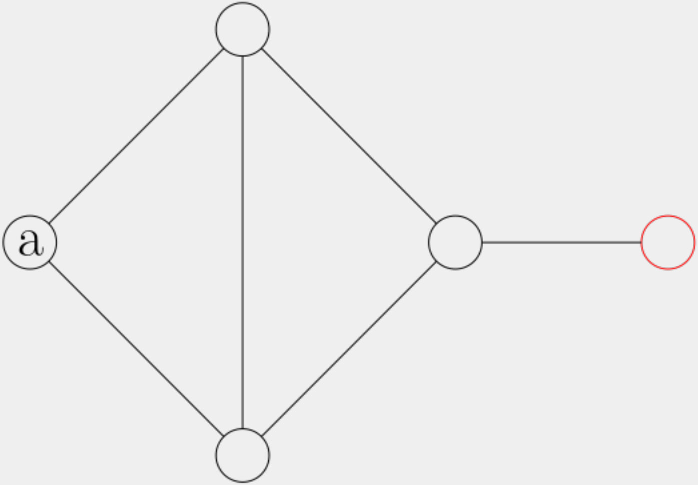
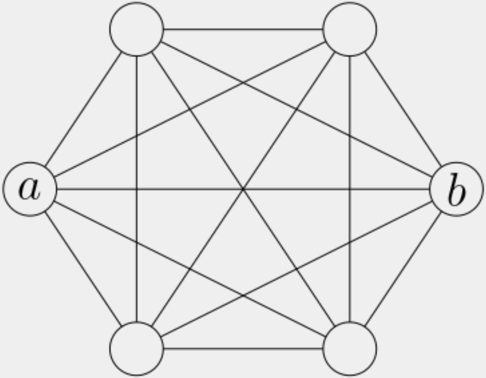


Let K_n be a complete graph, $n \geq 3$. Then, given two distinct vertices $a, b \in V(K_n)$, the expected hitting time of b is $n - 1$.



It is known that for every two vertices on any graph, the expected hitting time is not worse than $\mathcal{O}(n^3)$.

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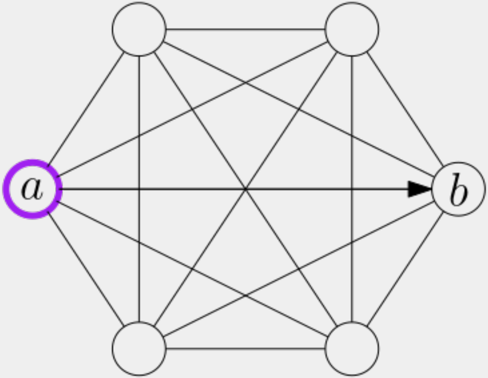
But what if we help the random walker in some way?

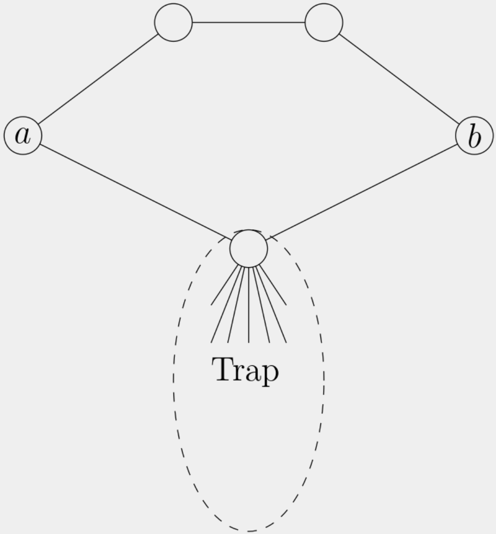
The main question

Given a graph $G = (V, E)$, choose some vertices $F \subseteq V$. In these 'excited' vertices, the random walker will deterministically take a step along a fixed shortest path.
Does the hitting time change, and if so, how?

Returning to the complete graph, if we excite a, the expected hitting time becomes 1.



Not so fast



- Can we show the same $\mathcal{O}(n^3)$ bound on the expected hitting time as before?

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- Can we show any polynomial bound?
- Are there any other natural 'biases', which help the random walker?

Acknowledgements

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