Fair Coloring of Planar Cubic Graphs

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Definition (Planar cubic graph)

We call a graph

- **planar** if it can be drawn in the plane without crossing, and
- **cubic** if the degree of all the vertices is three.
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Definition (Coloring)

A \textit{k-coloring} is an assignment of \textit{k} different colors to the vertices of the graph.

Definition (Proper coloring)

A coloring is \textit{proper} if no two adjacent vertices have \textit{the same color}.

Figure: Proper coloring

Figure: Non-proper coloring
**Definition (Coloring)**

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**Definition (Proper coloring)**

A coloring is **proper** if no two adjacent vertices have the same color.
**Theorem (Four color)**

*Every planar graph has a *proper* 4-coloring.*

**Definition (Fair coloring)**

A proper 4-coloring of a cubic planar graph is *fair* if all the neighbors of each vertex have *distinct* colors.

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**Figure: Fair coloring**

**Figure: Non-fair coloring**
**Theorem (Four color)**

*Every planar graph has a proper 4-coloring.*

**Definition (Fair coloring)**

A proper 4-coloring of a cubic planar graph is **fair** if all the neighbors of each vertex have **distinct** colors.

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**Figure:** Fair coloring

**Figure:** Non-fair coloring
Problem

For a given cubic planar graph we want to decide if there exists a fair coloring of the graph.

Question

How difficult is such problem? Does there exist an efficient algorithm?

- Fair coloring of cubic graphs is hard.
- Fair-like coloring of subcubic planar graphs is also hard.
Problem

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Observation

There exists no fair coloring for graph which contains the cycle $C_5$.

- By the pigeonhole principle at least one color is used twice.
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Does the following graph have a fair coloring?
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Any questions or suggestions?