



REU Program  
2017

**Quantum Cohomology Rings**

**Generated by**

**Gromov-Witten Invariants**

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*Supported by SAST-ATPAC*

# Backgrounds and Terminology



# Grassmannian



\* What is Grassmannian?

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Let  $k$  be an algebraically closed field, and let  $k^n$  be the vector space of column vector with  $n$  coordinates. Given a nonnegative integer  $m \leq n$ , the **Grassmann variety** is defined as a set by

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For example, the set of lines  $Gr(1,n+1)$  is **projective space**.

# Schubert Varieties

The slide features decorative wavy lines in the top right and bottom left corners. The top right lines are in shades of purple, red, and orange, while the bottom left lines are in shades of green, yellow, and red.

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Let  $X = \text{Gr}(m, n)$  denote a fixed Grassmannian and set  $G = \text{GL}(n)$ . We also let  $T \subset G$  be the **maximal torus** of diagonal matrices and  $B \subset G$  the **Borel subgroup** of upper triangular matrices.



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The orbits of the action of  $B$  on  $X$  are called the **Schubert cells** of  $X$ , and their closures are **Schubert varieties**.



# Gromov-Witten invariants



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# Gromov-Witten invariants



Gromov-Witten invariants  
& Schubert Varieties



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Define

Quantum Cohomology Rings

# Gromov-Witten invariants

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Gromov-Witten invariants  
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Facts

Nonnegative integer



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Quantum Cohomology Rings



# Gromov-Witten invariants

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Force

Quantum Cohomology Rings

Facts

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# Goals





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- ❖ Can describe what quantum cohomology ring is.



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- ❖ Provide properties of quantum cohomology rings by using Gromov–Witten invariants.



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- ❖ Can describe what quantum cohomology ring is.
- ❖ Can describe what Gromov–Witten invariant is.
- ❖ Provide properties of quantum cohomology rings by using Gromov–Witten invariants.
- ❖ Get some ideas about what is **“Geometry”**.



A black circular logo with a white border containing the text "REU Program 2017".

REU Program  
2017

A large white circle with a drop shadow containing the text "THANK YOU!".

**THANK YOU!**

References:

Anders S. Buch. Notes on Grassmannians. [Online] 2014 . Available from <https://www.semanticscholar.org/paper/Notes-on-Grassmannians-BUCH/>. [Accessed : May 2017].