REU Program 2017

Quantum Cohomology Rings

Generated by

Gromov-Witten Invariants

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Supported by SAST-ATPAC

Backgrounds and Terminology

Grassmannian

* What is Grassmannain?

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For example, the set of lines Gr(1,n+1) is **projective space**.

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The orbits of the action of B on X are called the **Schubert cells** of X, and their closures are **Schubert varieties**.

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Gromov-Witten invariants & Schubert Varieties



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Quantum Cohomology Rings

Gromov-Witten invariants & Schubert Varieties

Define

Quantum Cohomology Rings

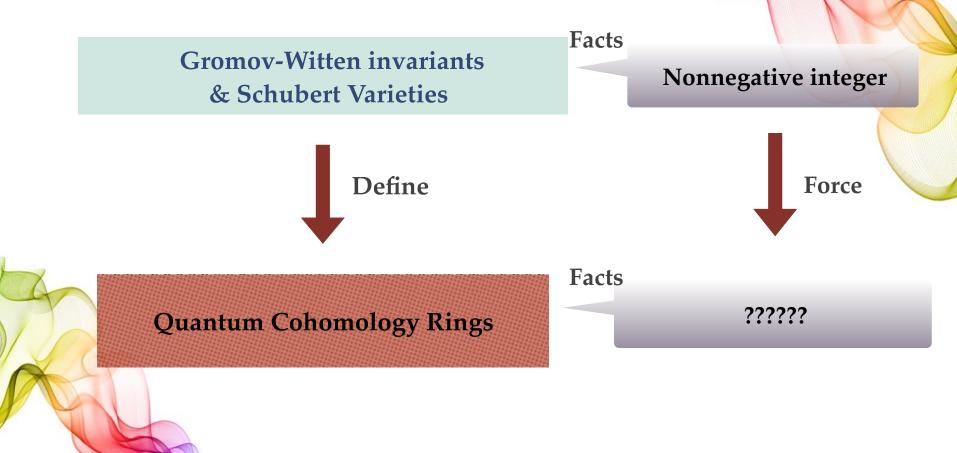
Facts

Gromov-Witten invariants & Schubert Varieties

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Define

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Can describe what quantum cohomology ring is.

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- **& Can describe what Gromov–Witten invariant is.**
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- **& Can describe what Gromov–Witten invariant is.**
- **Provide properties of quantum cohomology rings by using Gromov–Witten invariants.**
- ✤ Get some ideas about what is "Geometry".



THANK YOU!

References:

Anders S. Buch. Notes on Grassmannians. [Online] 2014 . Available from https://www.semanticscholar.org/paper/Notes-on-Grassmannians-BUCH/. [Accessed : May 2017].