On the Diophantine Equation $x^2 + D = k \cdot p^n$

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Outline

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- Our purpose

Diophantine Equation

Diophantine equation is an equation that has variable more than one. Its coefficients are integer and would like to find the solution in integer.

Example

1. $ax + by = c$ Linear equation
2. $x^2 + y^2 = z^2$ Pythagorean equation
3. $x^n + y^n = z^n$
4. $x^2 + D = 2^n$
5. $x^2 + D = kp^n$

F.Beukers proved that the diophantine equation

$$x^2 + D = 2^n,$$

where D is odd integer and $x, n \geq 1$, has at most four solutions for the case $D \neq 7$. For the case $D = 7$ we have the well-known Ramanujan-Nagell equation which has five solutions, namely,

$n = 3$	4	5	7	15
$x = 1$	3	5	11	181

Consider the diophantine equation

$$x^2 + D = A \cdot 2^n$$

where D is an odd integer, $A \geq 3$ a positive odd integer, $gcd(A, D) = 1$ and $x, n \geq 1$. For $D = 119$ and $A = 15$, Stiller proved that the equation has exactly six solutions

$n = 3$	4	5	6	8	15
$x = 1$	11	19	29	61	701

Our purpose

In our work, we will find that there is an absolute constant C such that the diophantine equation

$$x^2 + D = k \cdot p^n$$

has at most C solutions (x, n), where D and k be positive integers and p be a prime number such that $gcd(D, kp) = 1$.

Thank you
for your attention