Unique Rectification Targets in d-Complete Partial Orders

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Motivation

● K Theory
  ○ associates a number system (ring) with a geometric space
  ○ Ring illuminates properties of the geometric space

● Geometric Spaces: Flag Varieties

● Our project helps determine multiplication in these number systems
Partially Ordered Sets (Posets)

- Assigns hierarchy to elements in sets
- Ancestors of node are “less than” it
Skewed Poset Graph

- Remove “Order Ideal” from graph
  - Downwardly closed subset of graph

\[ G = \quad g = \quad G/g = \]
Rectification

- Process to turn a skew shape into a poset of straight shape (an order ideal)
- Algorithm is called Jeu De Taquin - tile game
  - Dot an empty node which is an “inner corner”
    - Inner corner means all of a node's children are filled
  - Swap dot with smallest child (if tie, swap both)
  - Delete dot node when leaf
  - Repeat until no more empty nodes
Rectification Example

- Begin with skewed graph
Rectification Example

- Fill external empty node with Dot
  - External = Node with children all filled
  - We have choice here
Rectification Example

- Swap dot with smallest child
Rectification Example

- Delete dot with no children
Rectification Example

- Add dot to inner corner
Rectification Example

- Swap dot with smallest child (if tie, swap both)
Rectification Example

- Delete dots with no children
Rectification Example

- Add dot to inner corner
Rectification Example

- Swap dot with smallest child
Rectification Example

- Delete dot with no children
Rectification Example

- Done!
Rectification Example

- Did our choices matter?
Order Matters?

- Didn’t matter that time
- In general can matter
Objective

Definition

A **Unique Rectification Target (URT)** is a labeling of an order ideal $P$ such that if some labeled skew poset rectifies to $P$, it will always rectify to $P$ no matter the choices you make during Jeu de Taquin.

Research Goal

Investigate the existence of URTs on a specific family of posets (d-Complete Posets)

First Result

Let $T$ be a tree. Then every labeling of every ideal of $T$ is a URT.
d-Complete Examples

Why d-complete?
- Lambda-minuscule varieties
Sources


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