Unique Rectification Targets in d-Complete Partial Orders

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Work supported by the Rutgers Math Department

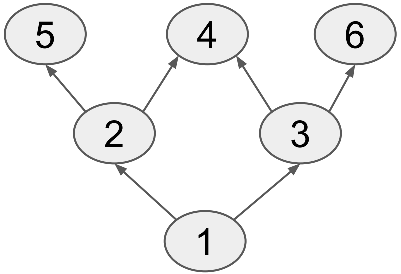
Background

Motivation

- K Theory and geometric spaces
  - associates a number system (ring) with a geometric space
  - Ring illuminates properties of the geometric space
- Our project helps determine multiplication in these number systems

Partially Ordered Sets (Posets / shapes)

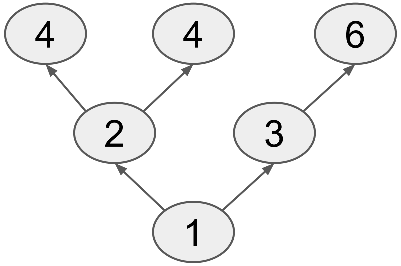
- Assigns hierarchy to elements in sets
- Ancestors of node are “less than” it



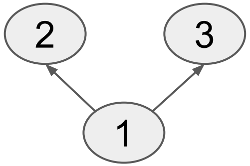
Skew shapes

- Remove downwardly closed subset

G =



g =



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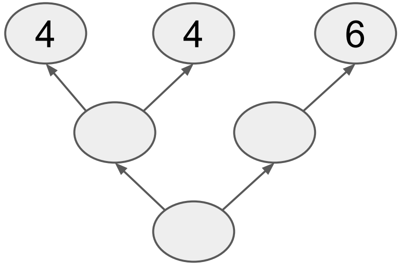
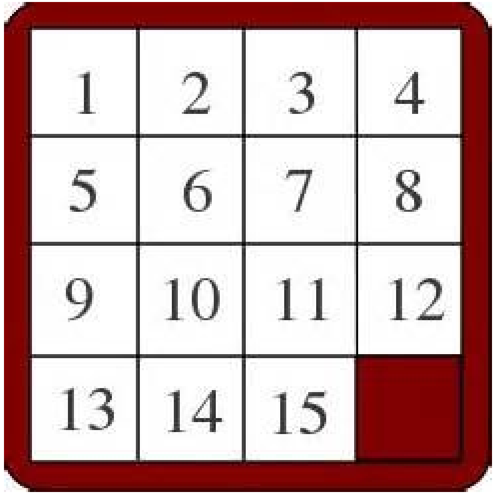


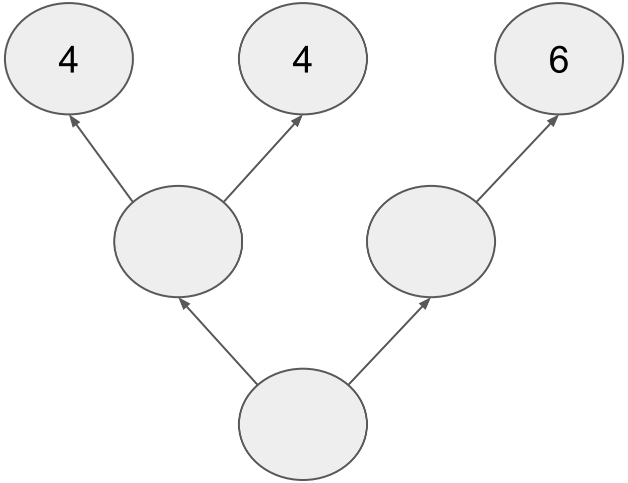
Rectification

- Process to turn a skew shape into a poset of straight shape (an order ideal)
- Algorithm is called Jeu De Taquin - tile game
  - Dot an empty node which is an “inner corner”
    - Inner corner means all of a node's children are filled
  - Swap dot with smallest child (if tie, swap both)
  - Delete dot node when leaf
  - Repeat until no more empty nodes



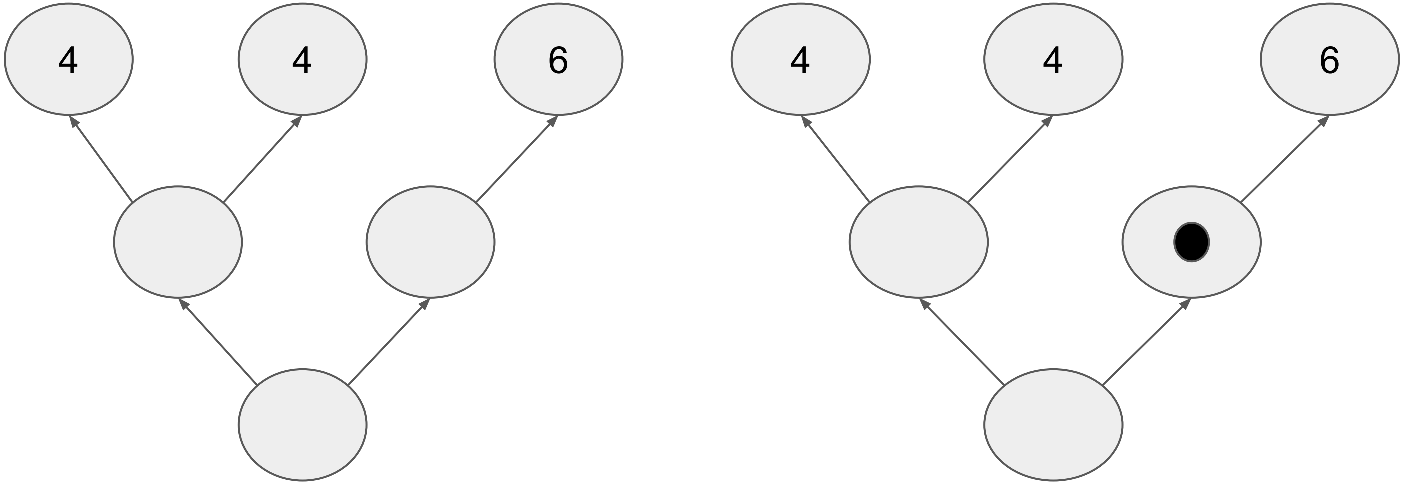
Rectification Example

- Begin with skewed graph



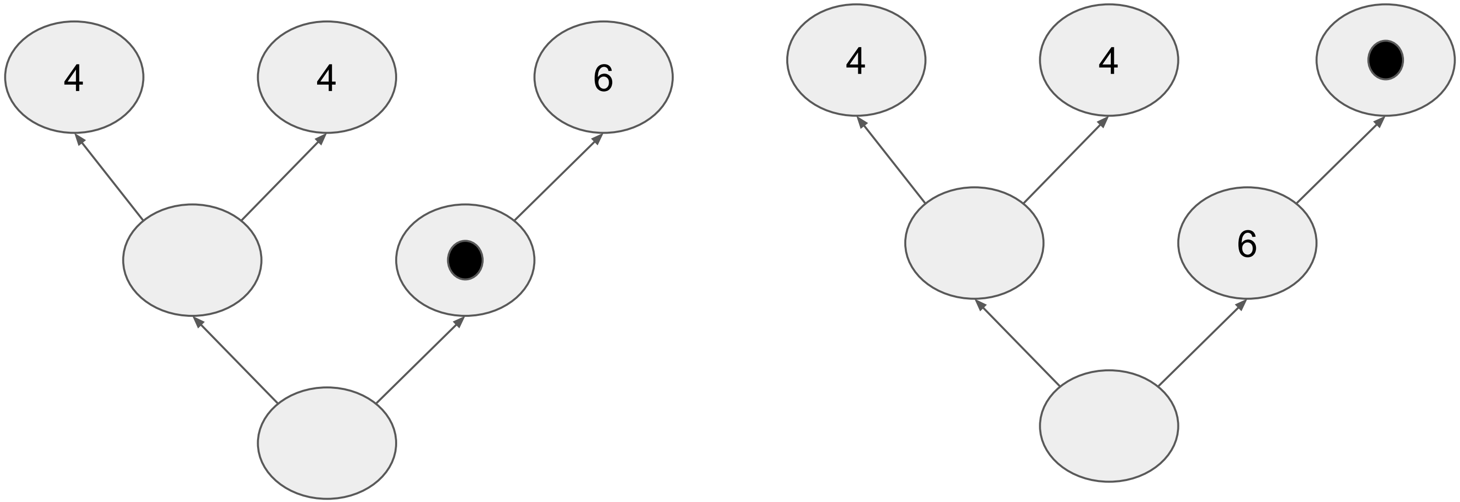
Rectification Example

- Fill external empty node with Dot
  - External = Node with children all filled
  - We have choice here



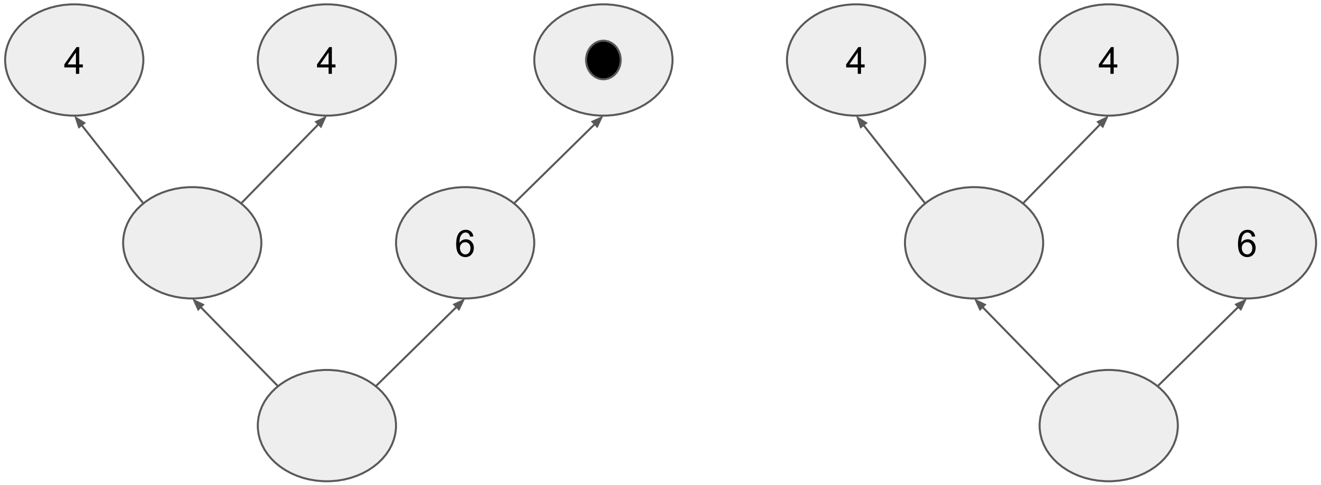
Rectification Example

- Swap dot with smallest child



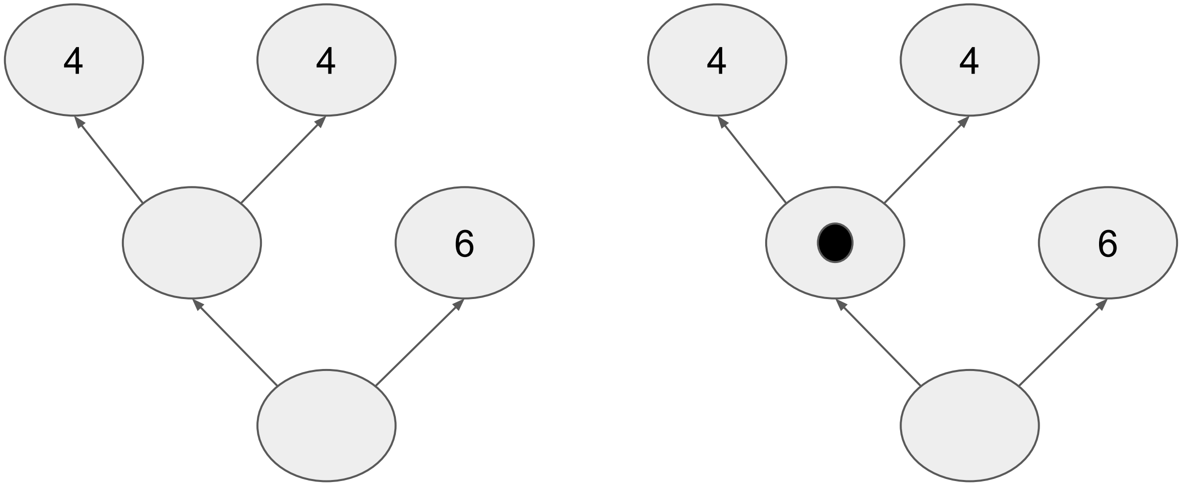
Rectification Example

- Delete dot with no children



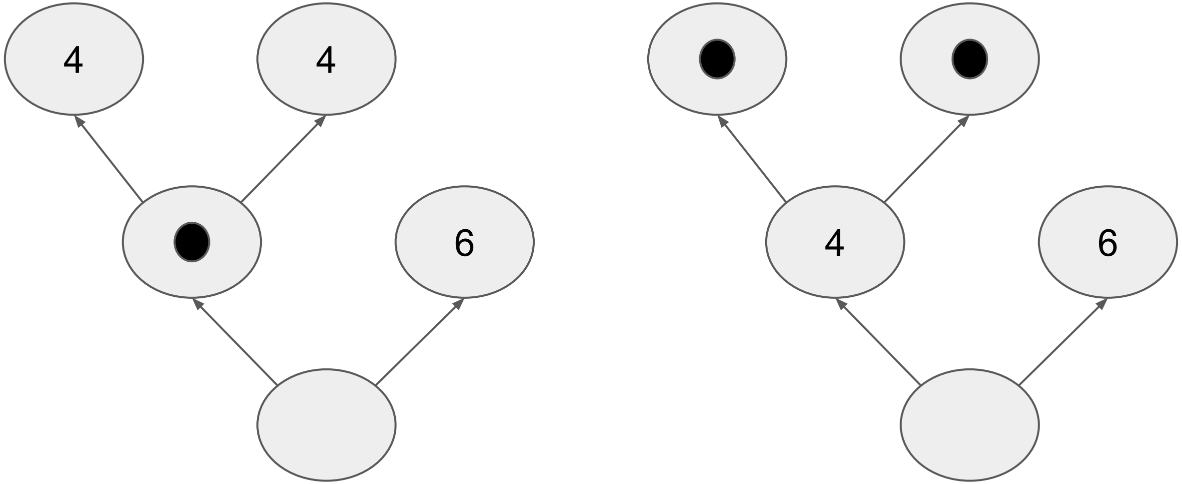
Rectification Example

- Add dot to inner corner



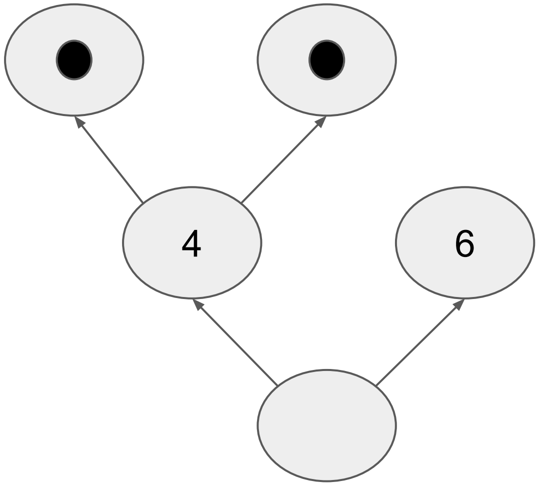
Rectification Example

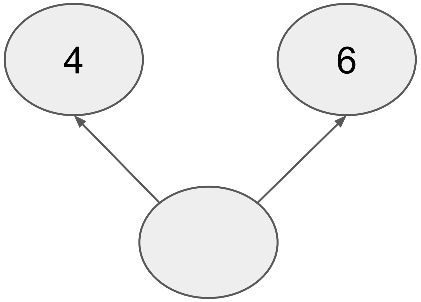
- Swap dot with smallest child (if tie, swap both)



Rectification Example

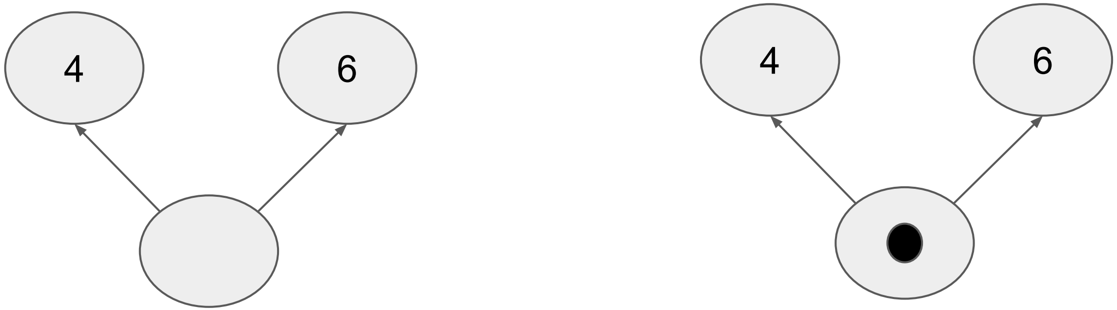
- Delete dots with no children





Rectification Example

- Add dot to inner corner

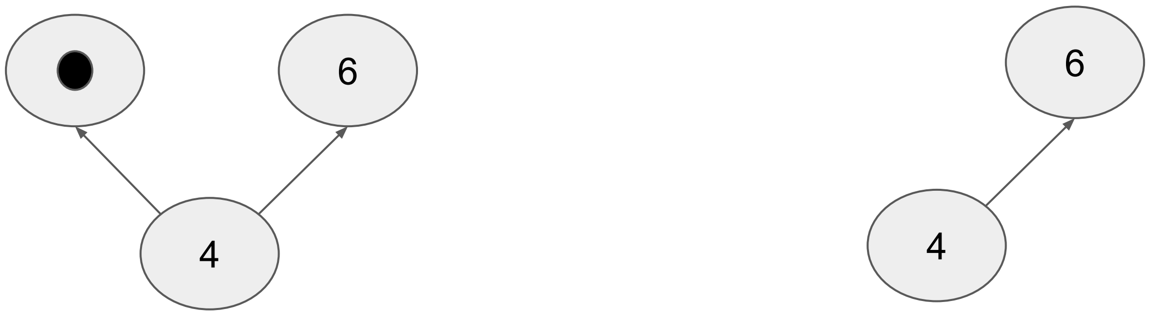


Rectification Example

- Swap dot with smallest child

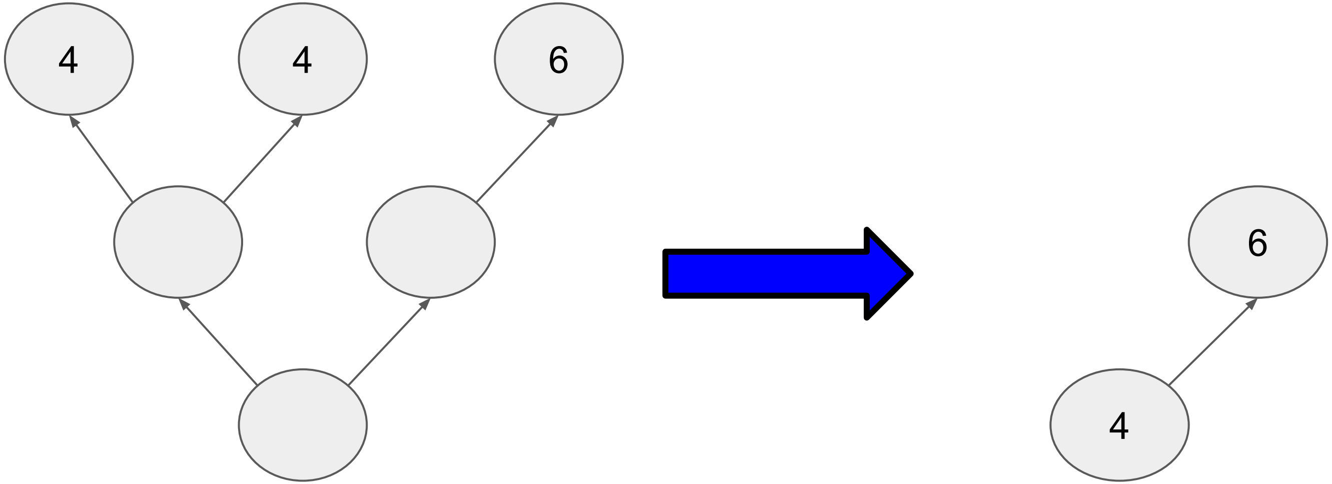
Rectification Example

- Delete dot with no children



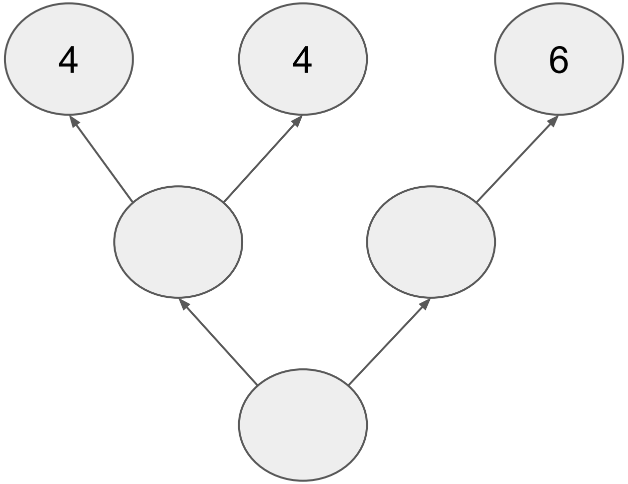
Rectification Example

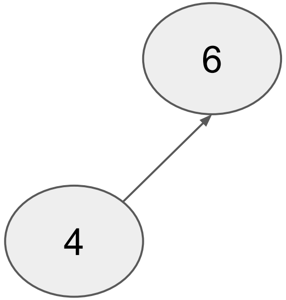
- Done!



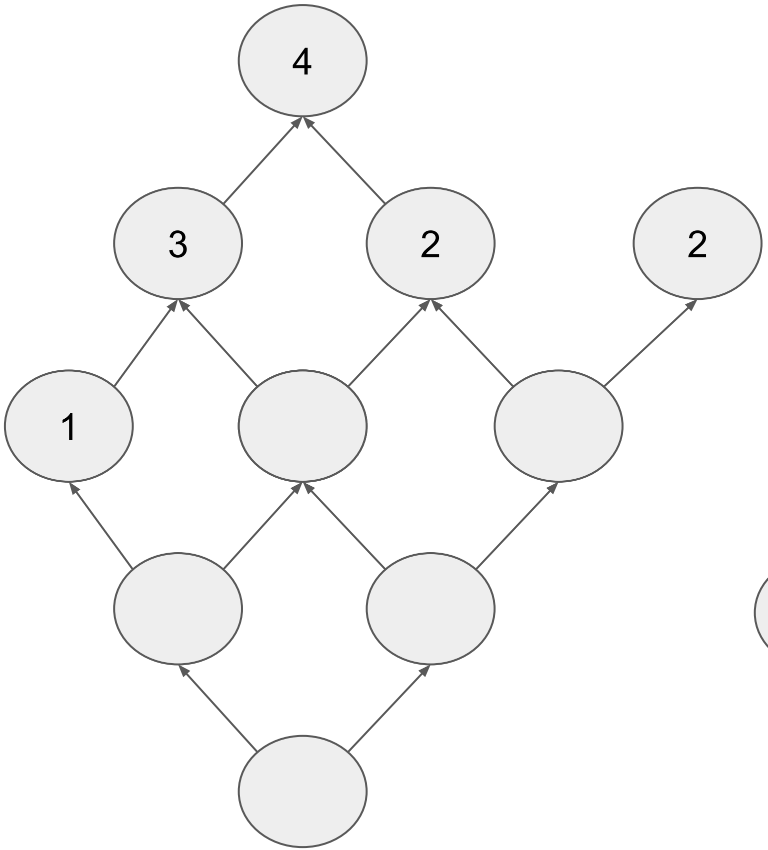
Rectification Example

- Did our choices matter?

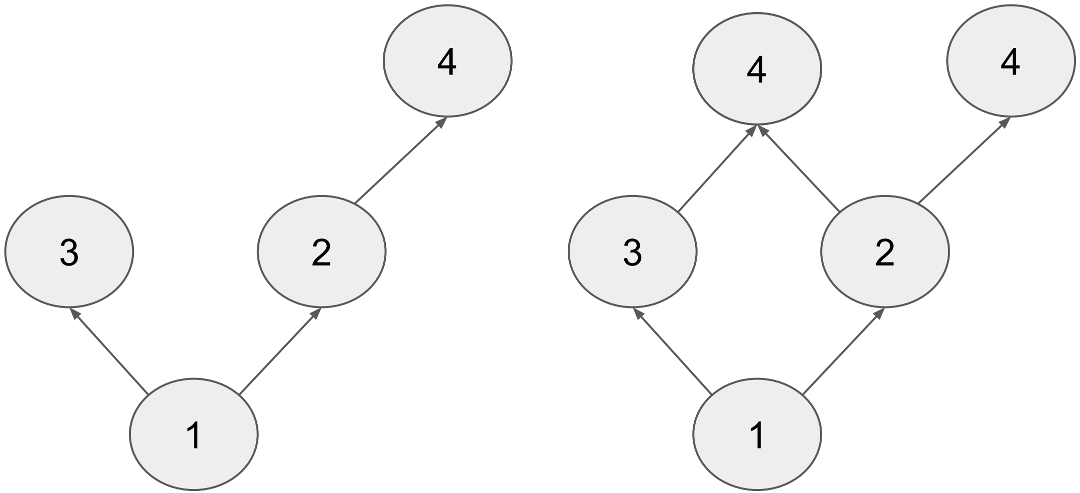




Order Matters?



- Didn't matter that time
- In general can matter



Objective

Definition

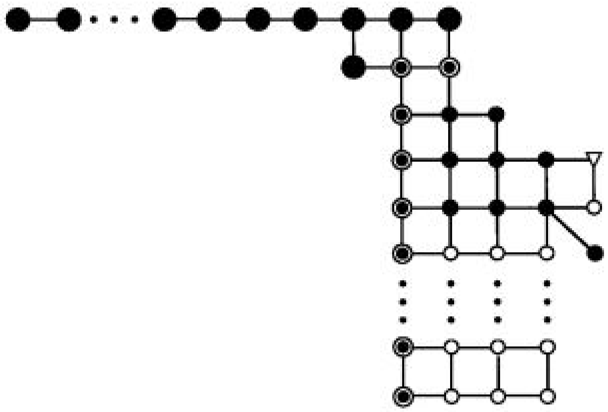
A Unique Rectification Target (URT) is a labeling T of an order ideal such that if some labeled skew poset rectifies to T, it will always rectify to T no matter the choices you make during Jeu de Taquin.

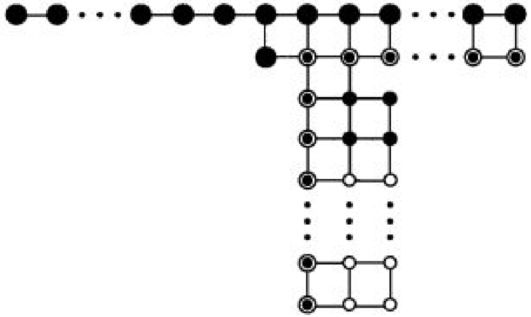
Research Goal

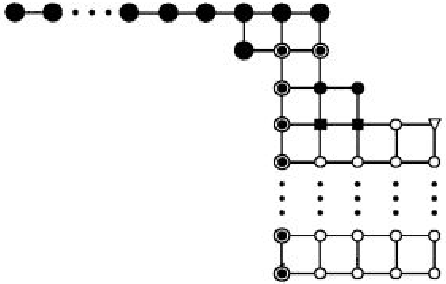
Investigate the existence of URTs on a specific family of posets (d-complete Posets)

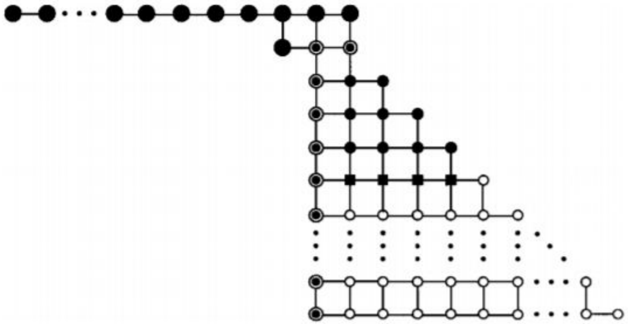
d-Complete posets

- Dictated by local constraints
- Composed of slant sum of “irreducible” posets



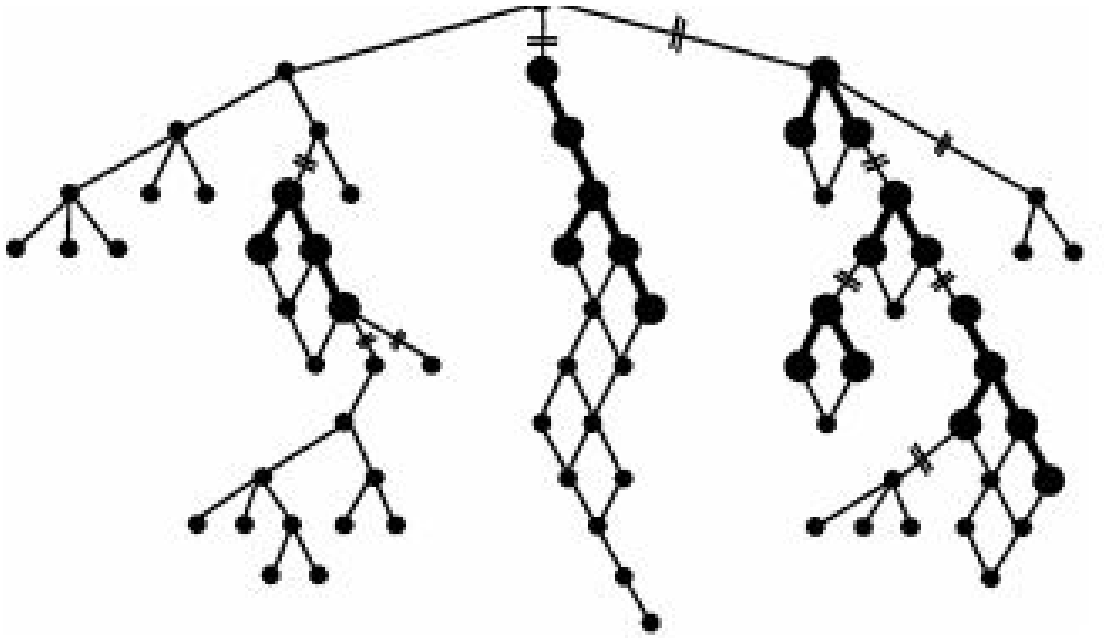






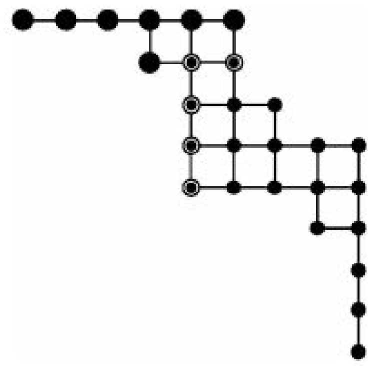
Slant sum

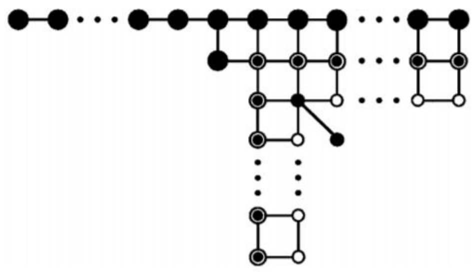
- Can attach d-Complete posets to each other at “acyclic” nodes

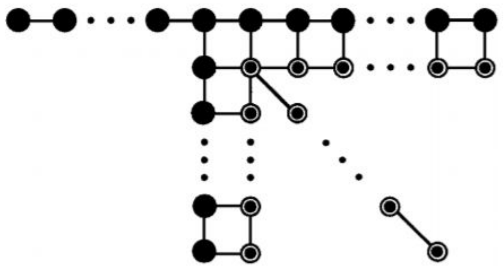


Why d-complete?

- Lambda-minuscule varieties
- Cohomology / standard labeling
- Seem to hold property empirically







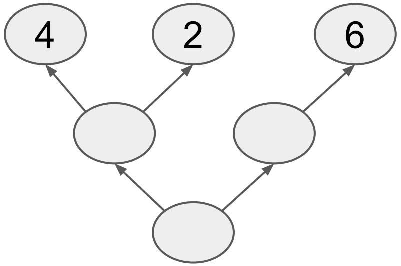
Ideas

- Look at URTs in each irreducible family
- See what we can say about how the slant sum operation preserves rectifications
- Computer

Results

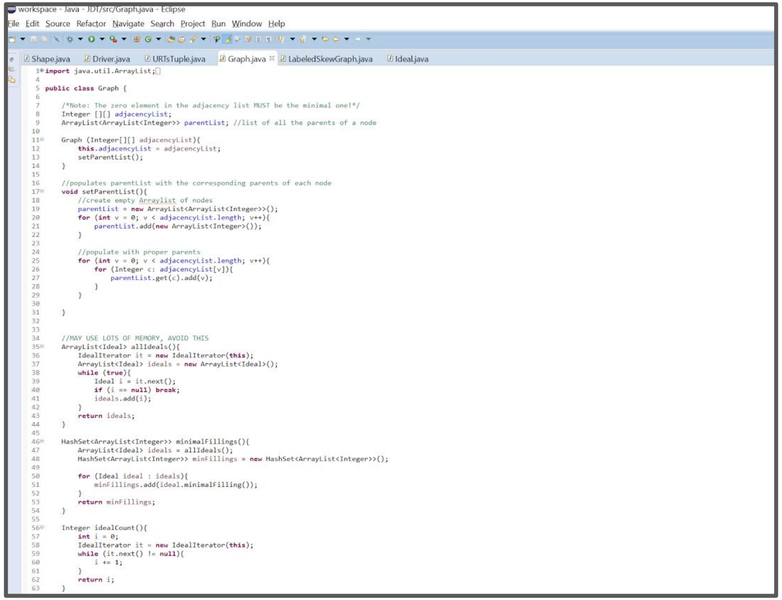
Trees

- In some sense, the "simplest" d-complete posets
- We prove that everything is a unique rectification target in trees

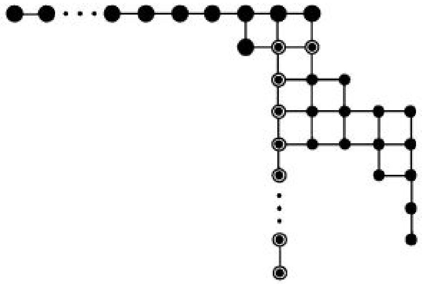


Computer Program

- Wrote computer program to find URTs in given posets
- Faster version checks for just minimal URTs

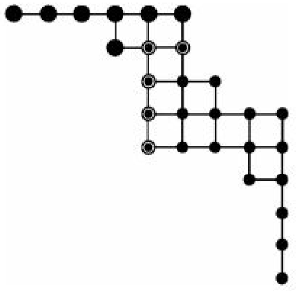


Adding “tails”



How does adding tails to elements affect a poset’s URTs?

- Adding below a minimum element preserve URTs
- Adding a new minimum below multiple posets preserves URTs
- Attaching posets to the nodes of a tree preserves URTs



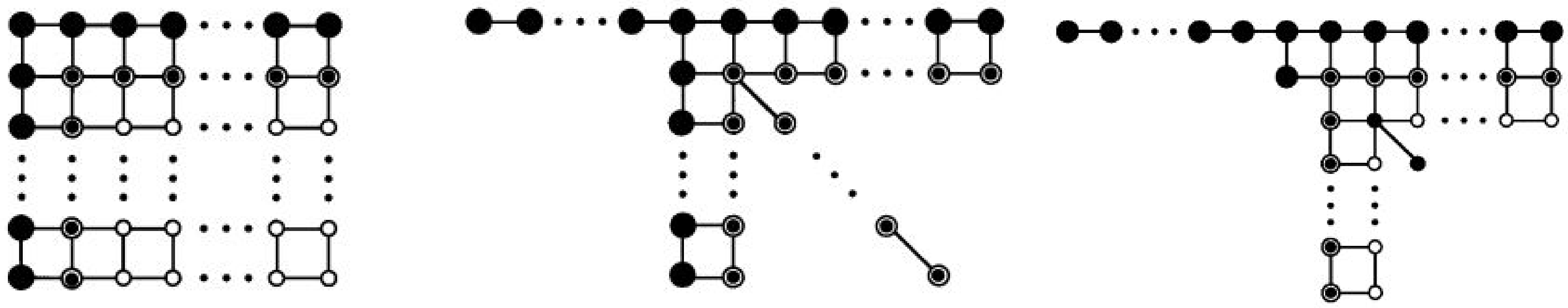
- Adding above elements does not preserve URTs in general

Slant Sum

- Adding above does not preserve all URTs
- We define p-chain URTs, which are exactly the URTs which are preserved when adding tails above
- p-chain URTs are preserved in slant sums (at a point p)

p-chain URTs in d-complete posets

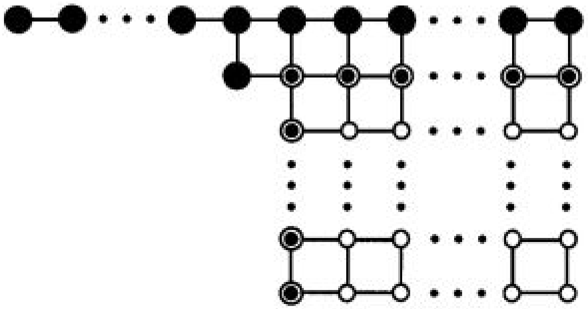
- 15 irreducible components
- 10 of these can be slant summed onto
- For 5 of these, we have shown that a poset is a URT in that shape if and only if it is a p-chain URT



Current Work

URTs in irreducible poset

- Work so far has concentrated on gluing together posets and d-complete posets in general
- Now we will focus in on individual posets and investigate their URTs and p-chain URTs
- Identify “reading word” which is slide invariant



Sources

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Acknowledgement

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