k-colored Point-set Embeddability of Graphs

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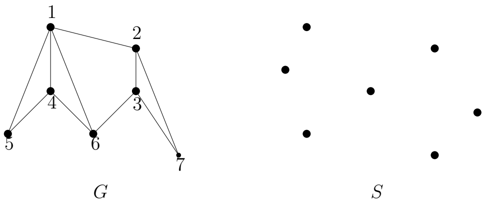
Point-set Embedding (PSE)

Definition

We are given a planar graph $G = (V, E)$ and a point set S $(|V| = |S|)$.



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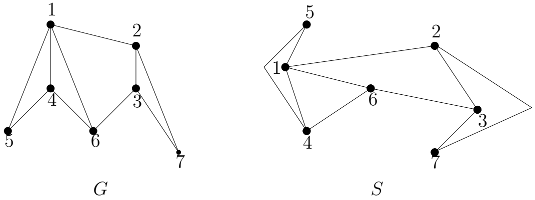
- A **point-set embedding** of G on S is a planar drawing such that each vertex is represented by a distinct point of S.



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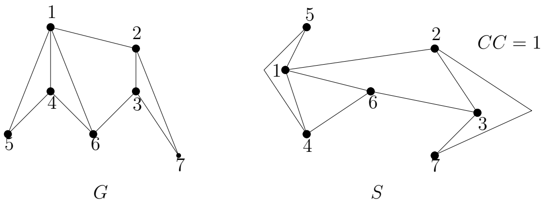
- A **curve complexity** (CC) of the PSE is the maximum number of bends along any edge.



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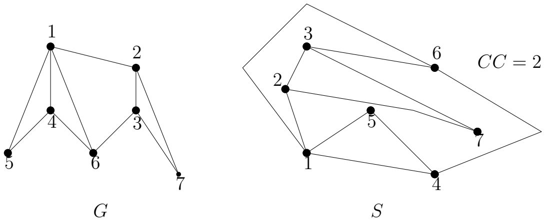
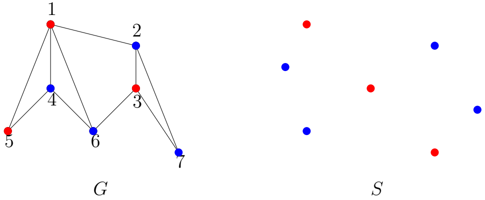


More Colors and More Graphs

Definition

In a **colored PSE**, both vertices and points are colored – a vertex can be represented **only** by a point of **the same color**.



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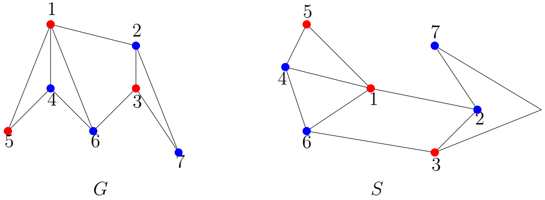

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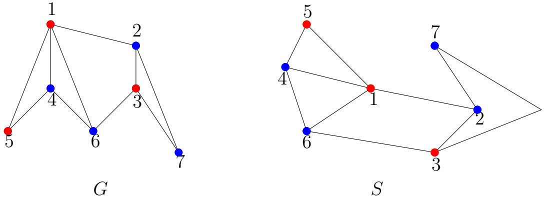


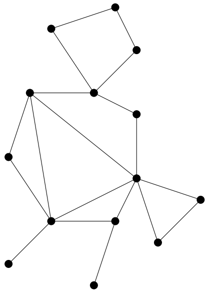
Problem

Given a **family** of planar graphs $\mathcal{G}$ and a number of colors k, we want to know the worst-case CC for any graph $G \in \mathcal{G}$ and any point set S.

Outerplanar Graph

Definition

An **outerplanar graph** is a graph for which there exists a planar drawing where all vertices belongs to the outer face.



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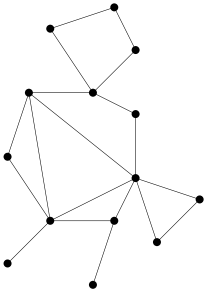

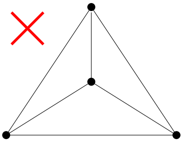

Known Results for Two Colors

	paths	caterp.	trees	outerpl.	planar
upper	1 [1]	1 [2]	5	5 [3]	$O(n)$
lower	1 [4]	1	1	1	$\Omega(n)$ [5]

[1] Di Giacomo, Liotta, Trotta IJFCS 2006; [2] Hančl pers.comm.; [3] Di Giacomo et al. JGAA 2008; [4] Kaneko, Kano, Suzuki TTGG 2004; [5] Badent, Di Giacomo, Liotta TCS 2008.

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Theorem (Our result)

Every 2-colored outerplanar graph can be embeded on any 2-colored compatible set with at most 4 bends per edge.

Augmenting Hamiltonian Cycle

- We use a technique described in [1].
- Let S be a k-colored sets of points in the plane. WLOG, we may assume that each point has a different x-coordinate. Otherwise, we slightly rotate the plane. Let $\sigma = \sigma_1, \ldots, \sigma_n$ denote the sequence of the points S according the x-coordinate.

[1] Di Giacomo et al. k-colored Point-Set Embeddability of Outerplanar Graphs. JGAA, 12(1) 29-49 (2008)

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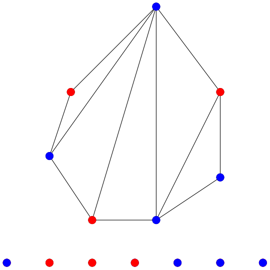

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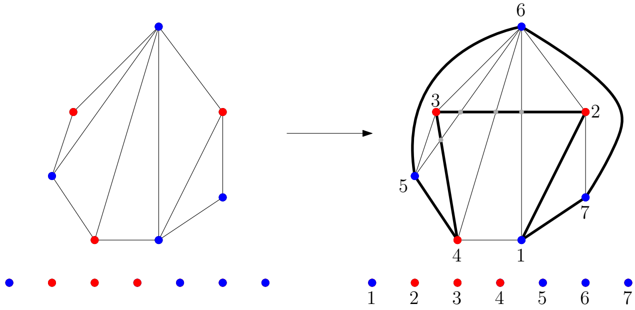

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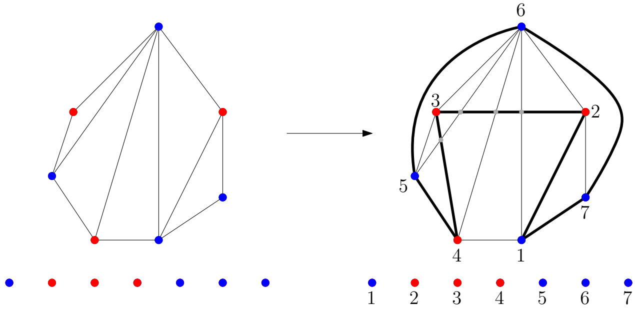

- Augmenting Hamiltonian circle **consistent with σ with at most d crossings per edge**

Augmenting Hamiltonian Cycle

Theorem [1]

Let G be a k-colored graph, let S be a k-colored set of points in the plane compatible with G and let σ be a sequence of points of S according to the x-coordinate. If G has an augmenting Hamiltonian circle consistent with σ with at most d crossings per edge then it can be embedded into S with at most $2d + 1$ bends per edge.

[1] Di Giacomo et al. k-colored Point-Set Embeddability of Outerplanar Graphs. JGAA, 12(1) 29-49 (2008)

Augmenting Hamiltonian Cycle

1. Add a division vertex of a new color for each crossing.
2. Add a dummy points for division vertices into S.
3. Embed such graph with 1 bend per edge.
4. Remove division vertices and obtain at most $(d+1)+d$ bends per edge.

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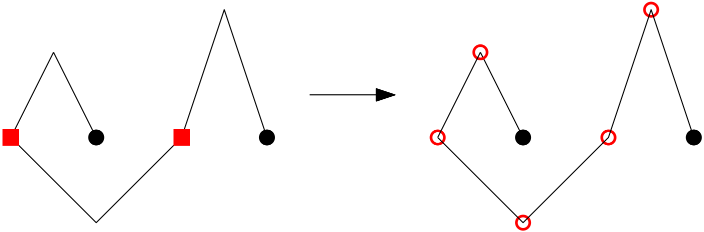

Improving the Theorem

Theorem

Let G be a k-colored planar graph, let S be a k-colored set of points in the plane compatible with G and let σ be a sequence of points of S according to the x-coordinate. If G has an augmenting Hamiltonian circle consistent with σ with at most d crossings per edge then it can be embedded into S with at most $2d$ bends per edge.

Using the Theorem for Outerplanar Graphs

Theorem [1]

Let G be an outerplanar 2-colored graph. Let S be a 2-colored set of points in the plane compatible with G and let σ be a sequence of points of S according to the x-coordinate. Then G has an augmenting Hamiltonian circle consistent with σ with at most 2 division crossings per edge.

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Corollary

Every 2-colored outerplanar graph can be embeded on any 2-colored compatible set with at most 4 bends per edge.

(Hopefully) Future Plans

Conjecture

Let G be an outerplanar 2-colored graph. Let S be a 2-colored set of points in the plane compatible with G and let σ be a sequence of points of S according to the x-coordinate. Then G has an augmenting Hamiltonian circle consistent with σ with at most 1 division crossing per edge.

(Hopefully) Future Plans

Conjecture

Let G be an outerplanar 2-colored graph. Let S be a 2-colored set of points in the plane compatible with G and let σ be a sequence of points of S according to the x-coordinate. Then G has an augmenting Hamiltonian circle consistent with σ with at most **1** division crossing per edge.

Corollary

Every 2-colored outerplanar graph can be embeded on any 2-colored compatible set with at most **2** bends per edge.

(Hopefully) Future Plans

Conjecture

Let T be a 2-colored tree. Let S be a 2-colored set of points in the plane compatible with G and let σ be a sequence of points of S according to the x-coordinate. Then T has an augmenting Hamiltonian circle consistent with σ **which is planar**.

(Hopefully) Future Plans

Conjecture

Let T be a 2-colored tree. Let S be a 2-colored set of points in the plane compatible with G and let σ be a sequence of points of S according to the x-coordinate. Then T has an augmenting Hamiltonian circle consistent with σ **which is planar**.

Corollary

Every 2-colored tree can be embeded on any 2-colored compatible set with at most **1** bends per edge.

Thank you!

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