# $k$-colored Point-set Embeddability of Graphs

Peter Korcsok and Michael Skotnica
mentor Periklis Papakonstantinou

Charles University, Prague

CHARLES UNIVERSITY
Faculty of mathematics
and physics

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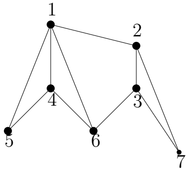
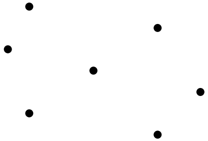
# Point-set Embedding (PSE)

## Definition

We are given a planar graph $G = (V, E)$ and a point set $S$ $(|V| = |S|)$.




$G$



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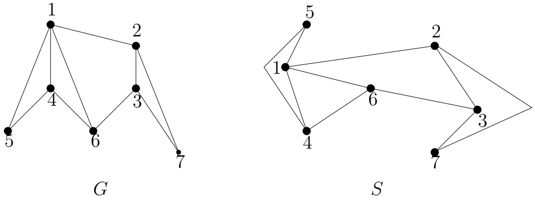
- A **point-set embedding** of $G$ on $S$ is a planar drawing such that each vertex is represented by a distinct point of $S$.



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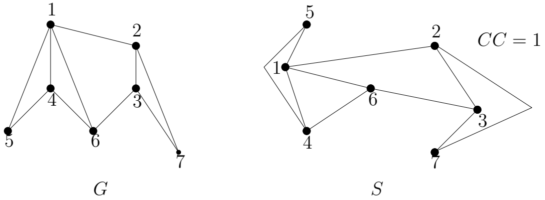
- A **point-set embedding** of $G$ on $S$ is a planar drawing such that each vertex is represented by a distinct point of $S$.
- A **curve complexity** (CC) of the PSE is the maximum number of bends along any edge.



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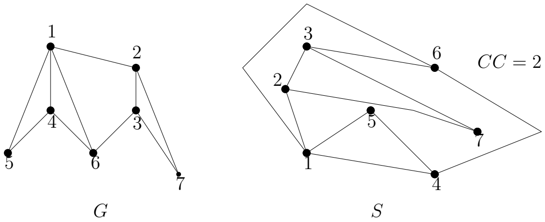
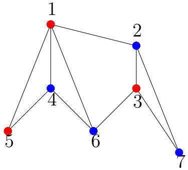


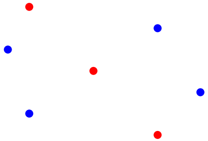
# More Colors and More Graphs

## Definition

In a **colored PSE**, both vertices and points are colored – a vertex can be represented **only** by a point of **the same color**.




$G$



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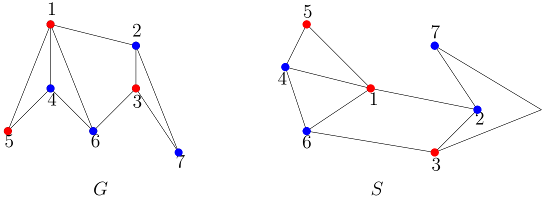

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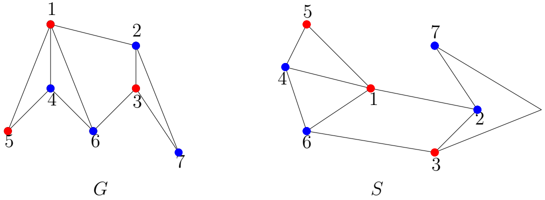


## Problem

Given a **family** of planar graphs $\mathcal{G}$ and a number of colors $k$, we want to know the worst-case CC for any graph $G \in \mathcal{G}$ and any point set $S$.

# Flow map drawing

- $k = 2$ colors
- $G$ is a rooted tree (root has its unique color and it represents Colorado), $S$ is a map of the United States

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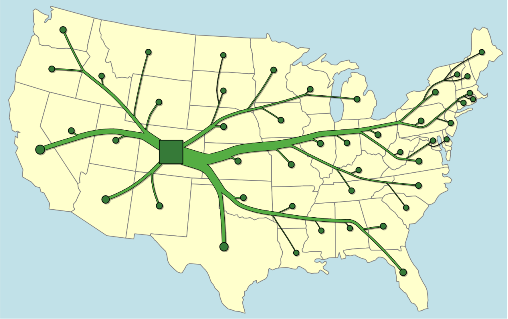

Migration from Colorado

[Verbeek, Buchin, Speckmann IEEE TVCG 2011]

# Known Results

**PSE: upper and lower bounds on CC**

| | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| ... | ... | ... | ... | ... | ... |
| n | | | | | |

[1] Badent, Di Giacomo, Liotta TCS 2008; [2] Bose CGTA 2002; [3] Di Giacomo et al. JGAA 2008; [4] Di Giacomo et al. GD 2017; [5] Di Giacomo, Liotta, Trotta IJFCS 2006; [6] Hančl pers.comm.; [7] Kaneko, Kano, Suzuki TTGG 2004; [8] Kaufmann, Wiese JGAA 2002; [9] Pach, Wenger G&C 2001

# Known Results

**PSE: upper and lower bounds on CC**

| | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|
| 1 | | | | | |
| | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | |
| 2 | | | | | |
| | | | | | |
| 3 | | | | | |
| | | | | | |
| ... | ... | ... | ... | ... | ... |
| n | | | | | |
| | | | | | |

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|---|---|---|---|---|---|
| 1 | | | | 0 [2] | |
| | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | |
| 2 | | | | | |
| | | | | | |
| 3 | | | | | |
| | | | | | |
| . . . | . . . | . . . | . . . | . . . | . . . |
| n | | | | | |
| | | | | | |

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**PSE: upper and lower bounds on CC**

| | | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|---|
| 1 | upper | 0 | 0 | 0 | 0 [2] | |
| | lower | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | |
| 2 | upper | | | | | |
| | lower | | | | | |
| 3 | upper | | | | | |
| | lower | | | | | |
| . . . | | . . . | . . . | . . . | . . . | . . . |
| n | upper | | | | | |
| | lower | | | | | |

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# Known Results

**PSE: upper and lower bounds on CC**

| | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 [2] | 2 [8] |
| | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 | | | | | |
| | | | | | |
| 3 | | | | | |
| | | | | | |
| . . . | . . . | . . . | . . . | . . . | . . . |
| n | | | | | |
| | | | | | |

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| | | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|---|
| 1 | upper | 0 | 0 | 0 | 0 [2] | 2 [8] |
| | lower | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 | upper | | | | | |
| | lower | | | | | |
| 3 | upper | | | | | |
| | lower | | | | | |
| . . . | | . . . | . . . | . . . | . . . | . . . |
| n | upper | | | | | $O(n)$ [9] |
| | lower | $\Omega(n)$ [9] | | | | |

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|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 [2] | 2 [8] |
| | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 | | | | | |
| | | | | | |
| 3 | | | | | |
| | | | | | |
| ... | ... | ... | ... | ... | ... |
| n | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ [9] |
| | $\Omega(n)$ [9] | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ |

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# Known Results

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| | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|
| 1 (upper) | 0 | 0 | 0 | 0 [2] | 2 [8] |
| 1 (lower) | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 (upper) | 1 [5] | 1 [6] | | 5 [3] | |
| 2 (lower) | 1 [7] | | | | |
| 3 (upper) | | | | | |
| 3 (lower) | | | | | |
| $\dots$ | $\dots$ | $\dots$ | $\dots$ | $\dots$ | $\dots$ |
| n (upper) | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ [9] |
| n (lower) | $\Omega(n)$ [9] | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ |

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| | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 [2] | 2 [8] |
| | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 | 1 [5] | 1 [6] | 5 | 5 [3] | |
| | 1 [7] | 1 | 1 | 1 | |
| 3 | | | | | |
| | | | | | |
| ... | ... | ... | ... | ... | ... |
| n | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ [9] |
| | $\Omega(n)$ [9] | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ |

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| | | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|---|
| 1 | upper | 0 | 0 | 0 | 0 [2] | 2 [8] |
| | lower | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 | upper | 1 [5] | 1 [6] | 5 | 5 [3] | $O(n)$ |
| | lower | 1 [7] | 1 | 1 | 1 | $\Omega(n)$ [1] |
| 3 | upper | | | | | |
| | lower | | | | | |
| … | | … | … | … | … | … |
| n | upper | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ [9] |
| | lower | $\Omega(n)$ [9] | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ |

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|---|---|---|---|---|---|---|
| 1 | upper | 0 | 0 | 0 | 0 [2] | 2 [8] |
| | lower | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 | upper | 1 [5] | 1 [6] | 5 | 5 [3] | $O(n)$ |
| | lower | 1 [7] | 1 | 1 | 1 | $\Omega(n)$ [1] |
| 3 | upper | 5 [4] | | | | |
| | lower | | $\Omega(\sqrt[3]{n})$ [4] | | $\Omega(\sqrt[3]{n})$ [3] | |
| ... | | ... | ... | ... | ... | ... |
| n | upper | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ [9] |
| | lower | $\Omega(n)$ [9] | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ |

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| 1 | upper | 0 | 0 | 0 | 0 [2] | 2 [8] |
| | lower | 0 [trivial] | 0 [trivial] | 0 [trivial] | 0 [trivial] | 2 [8] |
| 2 | upper | 1 [5] | 1 [6] | 5 | 5 [3] | $O(n)$ |
| | lower | 1 [7] | 1 | 1 | 1 | $\Omega(n)$ [1] |
| 3 | upper | 5 [4] | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |
| | lower | 1 | $\Omega(\sqrt[3]{n})$ [4] | $\Omega(\sqrt[3]{n})$ | $\Omega(\sqrt[3]{n})$ [3] | $\Omega(n)$ |
| ... | | ... | ... | ... | ... | ... |
| n | upper | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ [9] |
| | lower | $\Omega(n)$ [9] | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ | $\Omega(n)$ |

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# Our Plan

**PSE: upper and lower bounds on CC**

| | paths | caterp. | trees | outerpl. | planar |
|---|---|---|---|---|---|
| 1 | 0 / 0 [trivial] | 0 / 0 [trivial] | 0 / 0 [trivial] | 0 [2] / 0 [trivial] | 2 [8] / 2 [8] |
| 2 | 1 [5] / 1 [7] | 1 [6] / 1 | 5 / 1 | 5 [3] / 1 | $O(n)$ / $\Omega(n)$ [1] |
| 3 | 5 [4] / 1 | $O(n)$ / $\Omega(\sqrt[3]{n})$ [4] | $O(n)$ / $\Omega(\sqrt[3]{n})$ | $O(n)$ / $\Omega(\sqrt[3]{n})$ [3] | $O(n)$ / $\Omega(n)$ |
| ... | ... | ... | ... | ... | ... |
| n | $O(n)$ / $\Omega(n)$ [9] | $O(n)$ / $\Omega(n)$ | $O(n)$ / $\Omega(n)$ | $O(n)$ / $\Omega(n)$ | $O(n)$ [9] / $\Omega(n)$ |

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# Thank you!

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