Tighter Lower and Upper Bounds on Graph-Based Multi-Robot Path Planning

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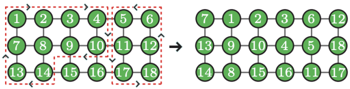
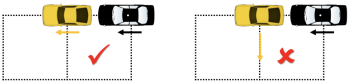
Multi-Robot Path Planning (MRPP) Problems

MRPP: Suppose you have a (grid) graph $G(V, E)$ and n robots with start configuration $S = \{s_1, \ldots, s_n\} \subseteq V$ and goal configuration $G = \{g_1, \ldots, g_n\} \subseteq V$. We seek to route the robots along collision-free paths P_i efficiently. In particular, we seek to minimize the overall makespan where motion is uniform without any meet or head-on collisions.

MRPP with CFC: Additionally, we may have a corner following constraint.



(a) Single time step move [1]

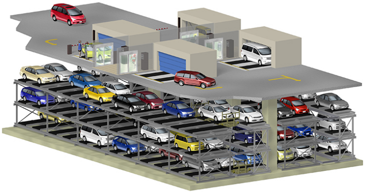


(b) Corner Following Constraint [2]

Example Applications

(a) Ocado Systems [a]

[a] "Inside a Warehouse Where Thousands Of Robots Pack Groceries." Tech Insider.



(b) Automated Garages [a]

[a] "Automated Multistory Parking Facilities." Trends in Japan.

Known Results

MRPP: Guo and Yu designed sub-1.5 time-optimal algorithms that run in polynomial time using Rubik Tables, which allowed arbitrary rearrangement of numbers in an $m_1 \times m_2$ grid using $2m_i + m_j$ row/column permutations [3].

MRPP with CFC: They were also able to apply Rubik Table results when the $m_1 \times m_2$ grid has $\theta(m_1 m_2)$ open spots, finding an $O(m_1 + m_2)$ makespan algorithm (upper bound) matching the $\Omega(m_1 + m_2)$ required makespan (lower bound). However, when there are only $\theta(m_1 + m_2)$ or $\theta(1)$ empty spots, with the same lower bound, they could only provide $O(m_1 m_2)$ makespan algorithms, even with Rubik Tables [2].

Figure: Rubik Table [1]

Research Questions

MRPP with CFC

- Suppose there are $\theta(m_1 + m_2)$ or $\theta(1)$ empty spots. Can we improve the required makespan lower bound for arbitrary rearrangement?
- On the other hand, can we use Rubik Tables or some alternative method to improve the solution makespan?

Additional Directions

- Extensions of MRPP Paper, e.g. life-long MRPP settings, more realistic robot models, better optimality at lower densities

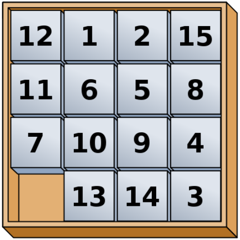


Figure: 15-Puzzle [1]. MRPP with CFC is a much more generalized version

[1] "15 Puzzle." Wikipedia.

Acknowledgements and References

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M. Szegedy and J. Yu, "Rubik tables and object rearrangement," 2023.

T. Guo and J. Yu, "Toward efficient physical and algorithmic design of automated garages," 2023.

T. Guo and J. Yu, "Sub-1.5 time-optimal multi-robot path planning on grids in polynomial time," 2022.