Embedding a 2D simplicial complex into 3D space

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Czech group of REU 2016

A familiar problem

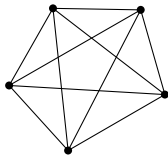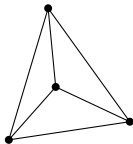### Problem

Given a simple graph, can you draw it into the plane such that no two edges intersect?

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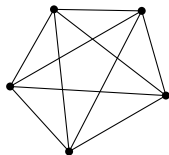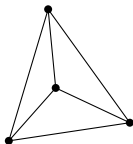You can draw $K_4, \ldots$ $\ldots$but you cannot draw K_5.

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If a drawing exists it can be found in linear time.

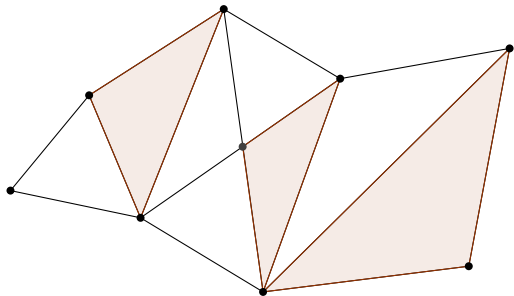# A generalization

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- 2D simplicial complex: 0D vertices, 1D edges and 2D triangles

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Basically a regular graph with some triangles filled.

Our task

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Can a given 2D simplicial complex be drawn into the 3D space such that no edges or triangles intersect, edges are drawn as straight line segments and triangles are flat?
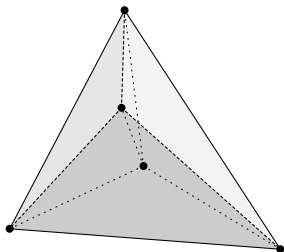
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K_5 with all triangles filled can be drawn, K_6 cannot be drawn.

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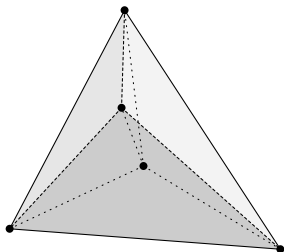

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Conjecture

Deciding whether such a drawing exists is an NP-hard problem.

Acknowledgement

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