Creating and Classifying an Alternate Hierarchy

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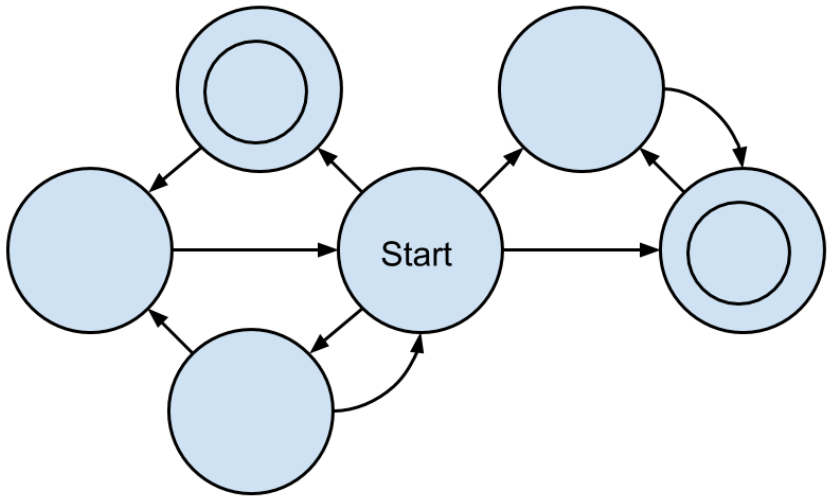
Deterministic Polynomial Time

Definition

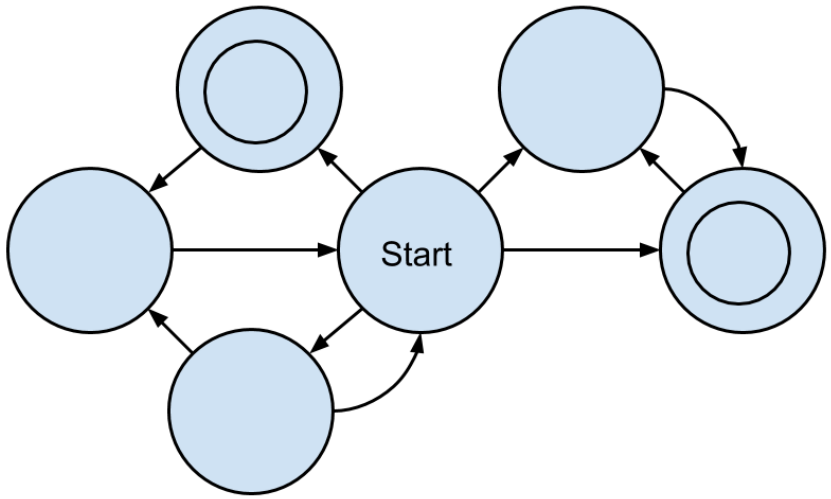
A language L is in **P** if for any $x \in L$, there is a polynomial time machine M such that $M(x) = 1$.



Nondeterministic Polynomial Time

Definition

A language L is in **NP** if for any $x \in L$, there is a polynomial time machine M such that $\exists a\, M(x, a) = 1$.



Nondeterministic Polynomial Time (Complement)

Definition

A language L is in **coNP** if for any $x \in L$, there is a polynomial time machine M such that $\forall a\, M(x, a) = 1$.

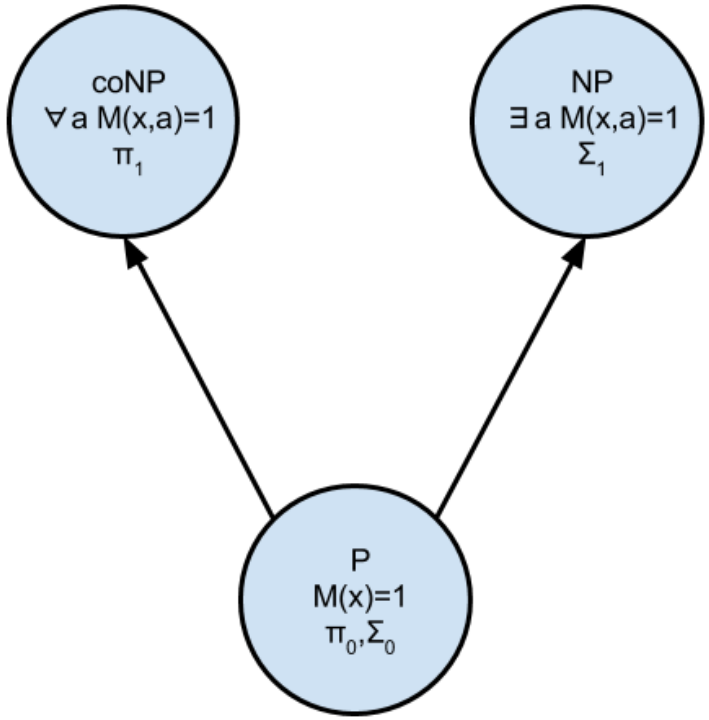
Note, the complement of an NP problem is:

$$\neg \exists a\, M_1(x, a) = 1,$$

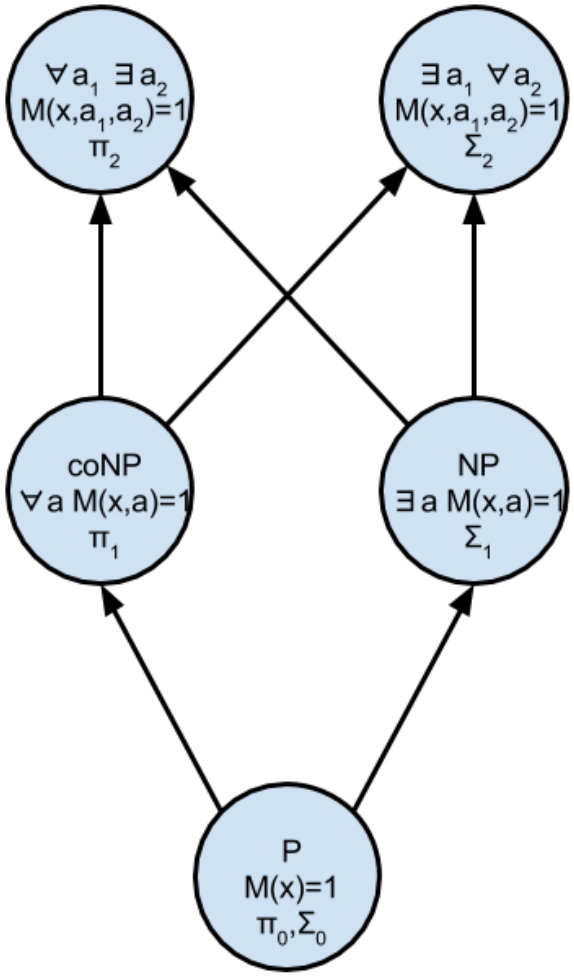
$$\forall a\, M_1(x, a) = 0,$$

$$\forall a\, M_2(x, a) = 1.$$

Polynomial Hierarchy

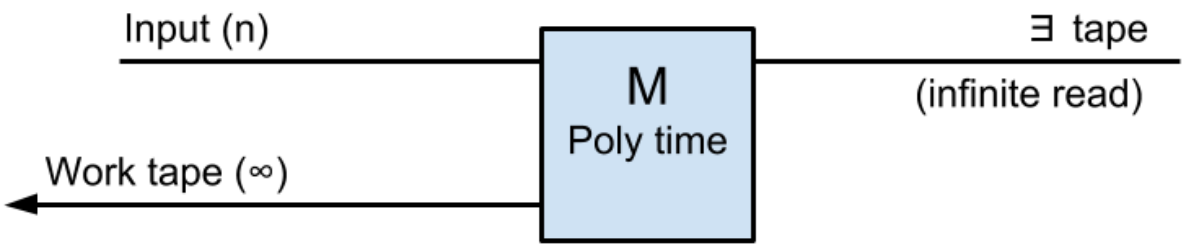


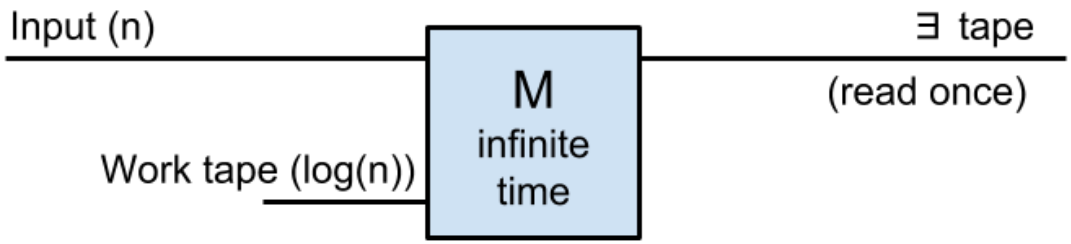
Polynomial Hierarchy



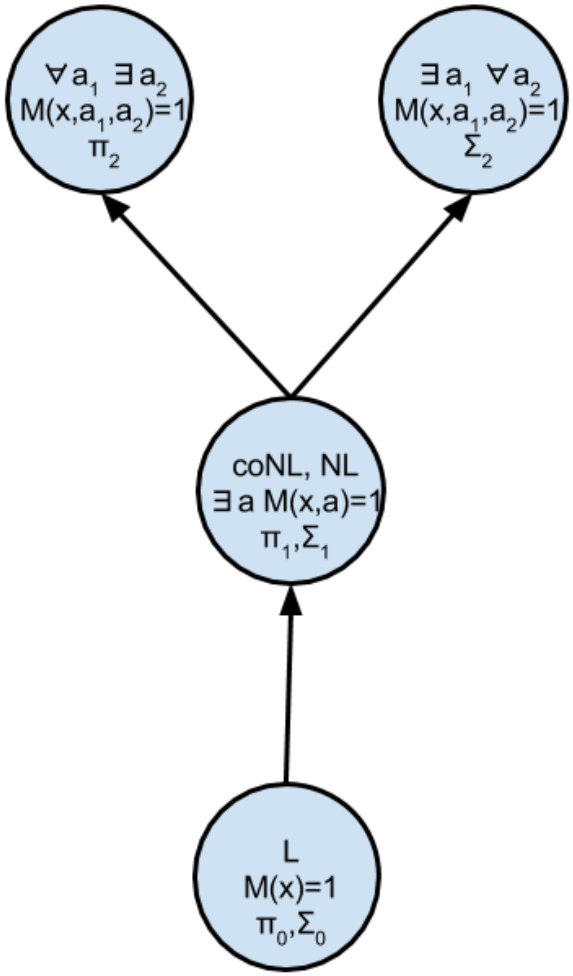
Defining a New Logspace Hierarchy

Polynomial machine vs our machine

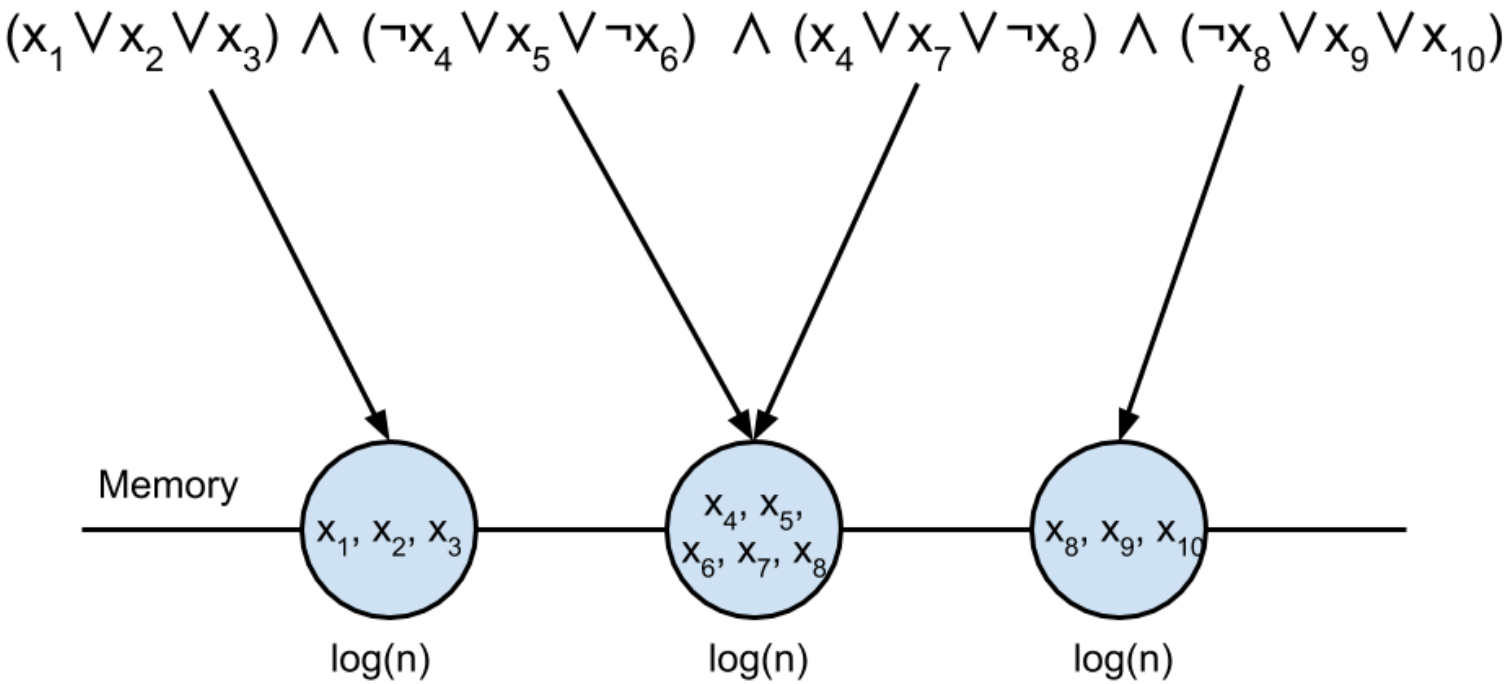




Defining a New Logspace Hierarchy



The Pathwidth Problem

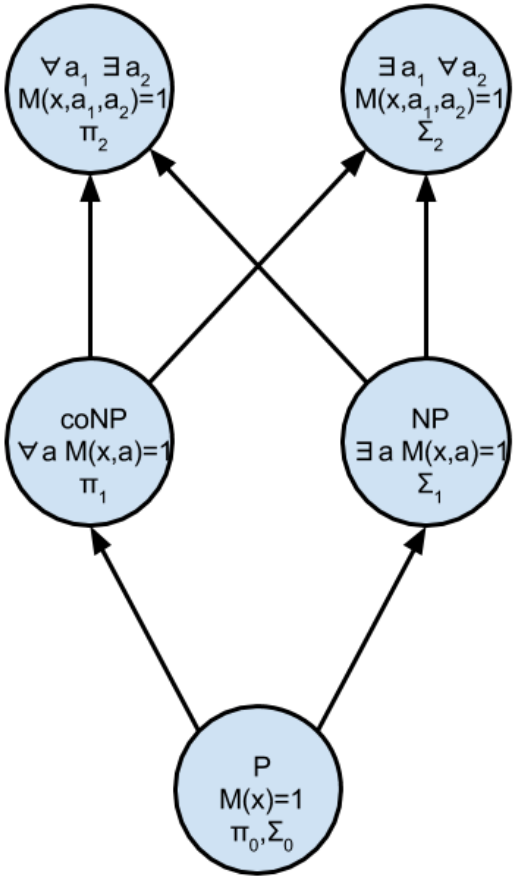


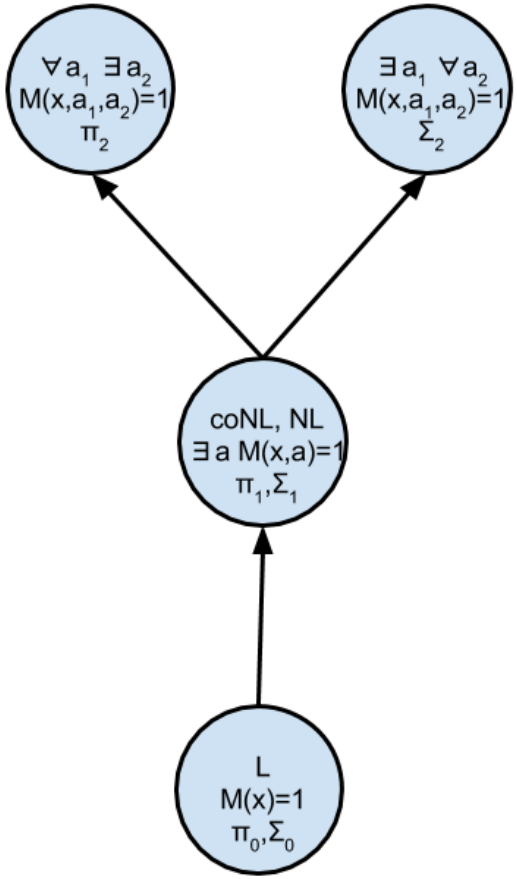
$\hat{\Sigma}_1^L$ Complete Problem

- Proof of completeness
  - Containment: clearly, M can solve bounded pathwidth SAT
  - Hardness: apply Cook-Levin Theorem to represent M as boolean expression, each step uses only logspace, so the expression is a bounded pathwidth SAT problem

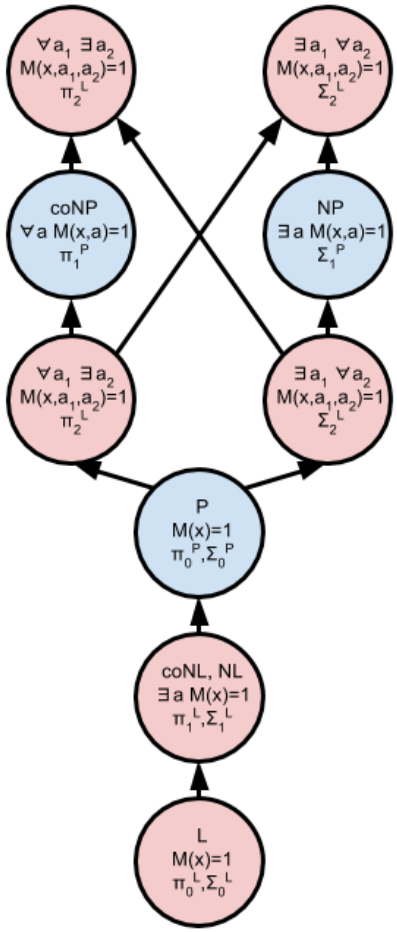
- This works for other levels of the hierarchy
  - ie, $\forall a_1 \exists a_2\, SAT$ with logarithmically bounded pathwidth is complete for $\hat{\Pi}_2^L$

Comparisons with the Polynomial Hierarchy





Combined Hierarchy



Conclusion

- We've created a new machine that creates a heirarchy that probably doesn't collapse
  - Else the polynomial hierarchy would collapse!
- We know a characterization for the complete problems for the entire hierarchy
- We know how it fits with the polynomial hierarchy
- But it isn't helpful to the Group Isomorphism problem (see: Czechs students' presentation)
- We hope to find more natural complete problems in the hierarchy

Acknowledgments

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Michael Garey and David Johnson: *Computers and Interactability: A Guide to the Theory of NP-Completeness*. WH Freemanx, 1979.