Using Kolmogorov Random Strings to Understand Complexity Class Relations

Student: Marina Knittel
Advisor: Professor Eric Allender

Harvey Mudd College

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A Brief Introduction to Reductions

- Reduction: an algorithm to solve a problem given the solution to another problem
- A reduces to B ($A \leq B$): there exists an efficient reduction from A to B
  - B can be used to solve A
  - B is at least as hard as A

Kolmogorov Complexity

- Kolmogorov Complexity $[K(x)]$: The length of the shortest program that prints its input, x
  - 10101011 can be written as $(10)^3 11$
- Kolmogorov random strings: $R_K = \{x \mid K(x) \geq |x|\}$
  - "Simplest" form strings
  - Not random: $10101011 = (10)^3 11$
  - Probably random: 28384329
- Notable reductions
  - $PSPACE \subseteq P^{R_K}$
  - $BPP \subseteq P_{tt}^{R_K}$

Promise Problems

- Given: A set of inputs L, and two sets $Y, N \subseteq L$ such that $Y \cap N = \emptyset$
- Goal: Accept everything in Y, reject everything in N, don't care on all other inputs
- Modification of R_K
  - L is the set of all possible input strings
  - $Y = \{x \mid K(x) \geq |x|\}$
  - $N = \{x \mid K(x) < g(|x|)\}$ where $g(|x|) \leq |x|$
  - Don't care about x such that $g(|x|) \leq K(x) < |x|$

My REU Project

- Explore the R_K promise problem and other complexity classes (specifically BPP)
- Don't know much about efficient reductions from BPP to R_K
- Anything reducible to the R_K promise problem is reducible to R_K

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