Bilu-Linial Conjecture and Ramanujan Graphs

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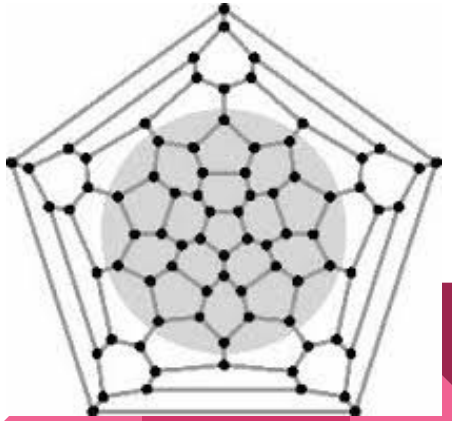
The Conjecture:

The Bilu-Linial conjecture claims every d-regular graph has a 2-lift such that all new eigenvalues are in the range

$[-2\sqrt{d-1}, 2\sqrt{d-1}]$

→ if this works can make Expander graphs



Marcus, Adam, Daniel Spielman, and Nikhil Srivastava. "Interlacing Families I: Bipartite Ramanujan Graphs of All Degrees." *Ann. Math. Annals of Mathematics* (2015): 307-25. Web.

Methods

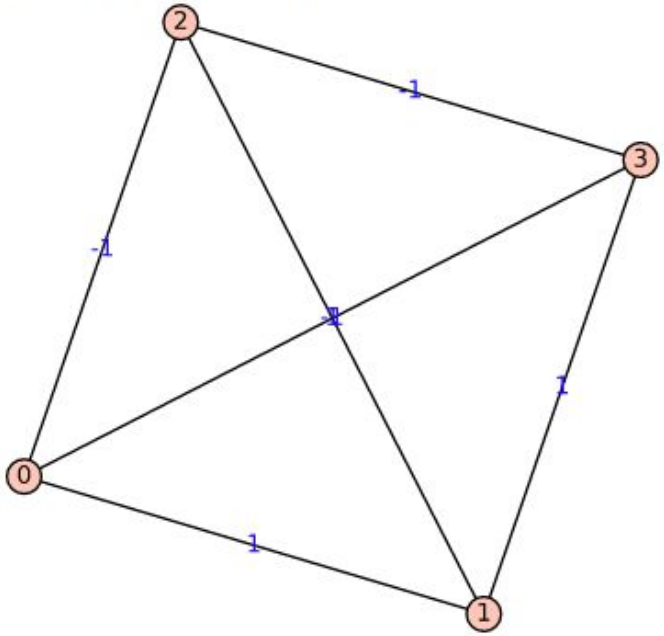
Wrote a program using SageMath software

- Iterates through all possible graphs on a specified number of vertices
- 2^E possible signed graphs, E = number of edges
- Verified the conjecture up to 10 vertices

Example Output

https://cloud.sagemath.com/projects/c33eaa71-79f7-4d70-81f0-bd207d9bb4ce/files/correct_signings.sagews

```
signing:
[1, -1, 1, -1, 1, -1]
```




```
signed Adjacenecy Matrix:
[ 0  1 -1  1]
[ 1  0 -1  1]
[-1 -1  0 -1]
[ 1  1 -1  0]
spectrum:
[3, -1, -1, -1]
```

Obstacles

Output is MUCH bigger than expected

- Computation Time
- Analysis

```
[-1, 1, -1, -1, -1, 1, -1, -1, -1, 1, -1, 1, -1, -1]
[1, 1, -1, -1, -1, 1, -1, -1, -1, 1, -1, 1, -1, -1]
[-1, -1, 1, -1, -1, 1, -1, -1, -1, 1, -1, 1, -1, -1]
[1, -1, 1, -1, -1, 1, -1, -1, -1, 1, -1, 1, -1, -1]
Output too long: 200073 -- MAX_OUTPUT (=200000) exceeded (type 'smc?' to learn how to raise this limit): attempting to terminate...
```

Next Steps

- Improve efficiency of program
- Nauty and Traces: programs used for computing automorphism groups of graphs and digraphs
  - B. D. McKay and A. Piperno, Practical Graph Isomorphism, II, *J. Symbolic Computation* (2013) **60** 94-112.
- Work on identifying patterns in the correct signings output
- Continue to work towards a proof of the conjecture and share my program with the community

Thank You

Rutgers Math Department

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DIMACS REU program

Director Lazaros Gallos