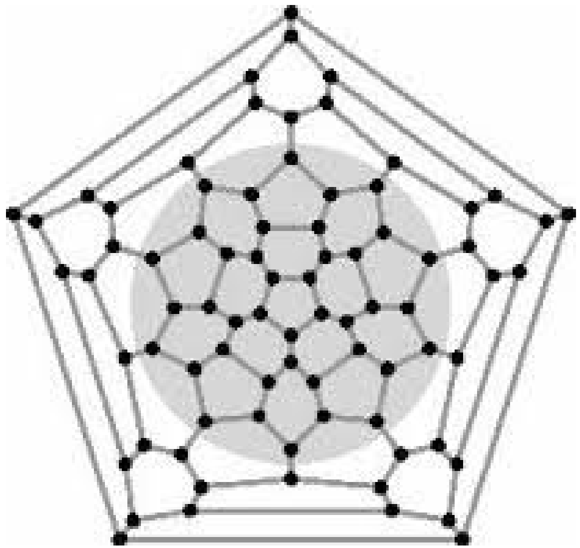
DIMACS

Center for Discrete Mathematics & Theoretical Computer Science
Founded as a National Science Foundation Science and Technology Center

Bilu-Linial Conjecture and Ramanujan Graphs

Marielle Jurist
Mentor: Ameera Chowdhury

Expanders and Applications



Expander graph = sparse but well connected

Applications in Computer science

- Generate expanders with same properties as large graphs like FB
- Can build infinite families of expander graphs using Ramanujan 2-lift

The Conjecture:

The Bilu-Linial conjecture claims every d-regular graph has a

2-lift such that all new eigenvalues are in the range

$[-2\sqrt{d-1}, 2\sqrt{d-1}]$

→ if this works can make Expander graphs

Background

What is a graph?

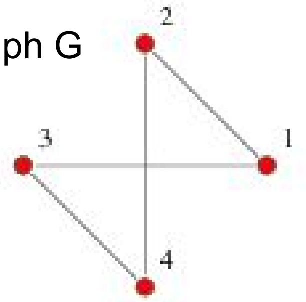
an ordered pair $G = (V, E)$ comprising a set V of *vertices* together with a set E of *edges*

From Adjacency Matrix of Graph G we can find Eigenvalues of G

This problem works with d-regular graphs

d-regular = all vertices have the same degree

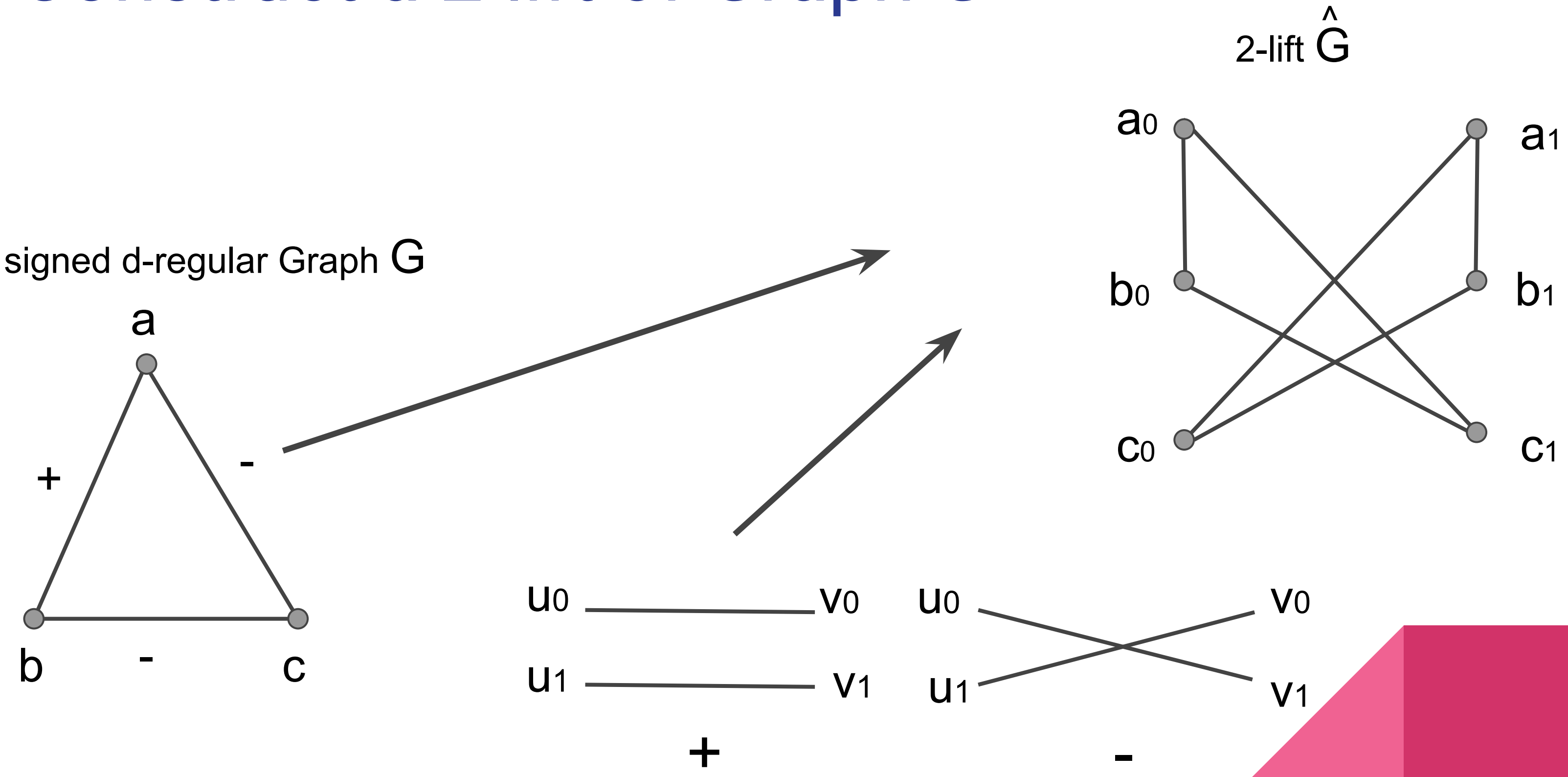
Graph G



Adjacency Matrix of G

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Construct a 2-lift of Graph G



Eigenvalues of $\hat{G}$ = Eigenvalues of G
$\cup$ Eigenvalues of A_s

$$
\begin{array}{c|cccccc|}
 & a_0 & b_0 & c_0 & a_1 & b_1 & c_1 \\
\hline
a_0 & 0 & 1 & 1 & 0 & 0 & 1 \\
b_0 & 1 & 0 & 0 & 0 & 0 & 1 \\
c_0 & 0 & 0 & 0 & 1 & 1 & 0 \\
a_1 & 0 & 0 & 1 & 0 & 1 & 1 \\
b_1 & 0 & 0 & 1 & 1 & 0 & 0 \\
c_1 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}
$$

Adjacency Matrix of $\hat{G}$

$$
\begin{array}{c|ccc|}
 & a & b & c \\
\hline
a & 0 & 1 & -1 \\
b & 1 & 0 & -1 \\
c & -1 & -1 & 0
\end{array}
$$

Signed Adjacency Matrix A_s

There exists a signing such that all the eigenvalues of A_s are in the range $[-2\sqrt{d-1}, 2\sqrt{d-1}]$