How do Cusped Surfaces Wear Pants?

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DIMACS

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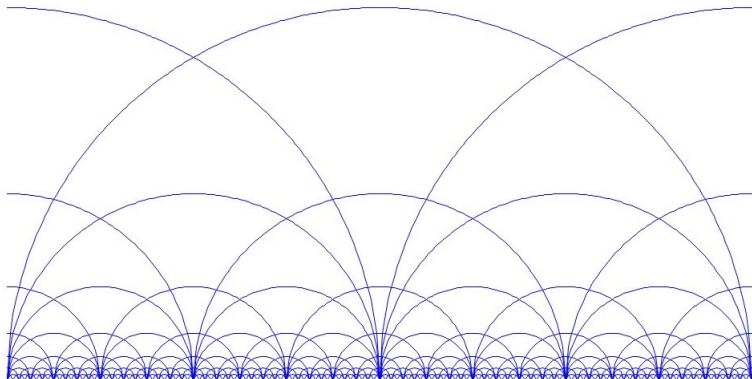
Motivation

- The uniformization theorem says that all (connected Riemann) surfaces of genus $g > 1$ have $\mathbb{H} = \{z \in \mathbb{C} : \Im(z) > 0\}$ as a universal covering space.

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- This cover does not necessarily appear as a finite cover for the surface. So, what can be said about existence of finite coverings?
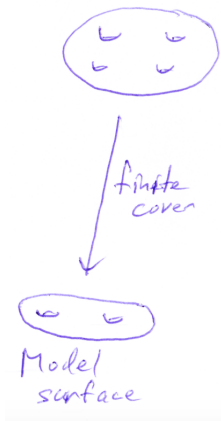
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[1] image from https://thatsmaths.files.wordpress.com/2013/10/halfplane1.jpg

finite cover

Model surface

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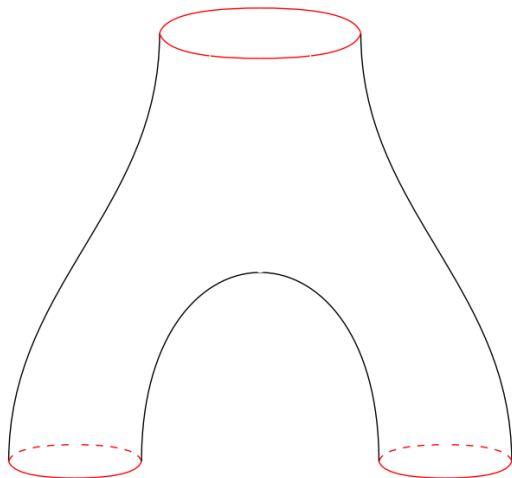
The Ehrenpreis conjecture

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- What can be said about surfaces with cusps?

The Ehrenpreis conjecture

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- The Ehrenpreis conjecture was proven true by Jeremy Kahn and Vladimir Markovic in 2011 using immersed pairs of pants to construct covers.
- What can be said about surfaces with cusps?
- Conjecture: Given any two cusped (Riemann) surfaces of genus $g > 1$, there exists finite covers which are "almost the same."
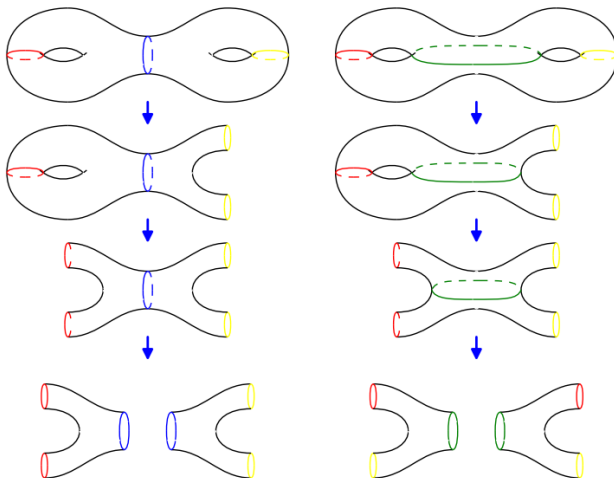
Pair of Pants

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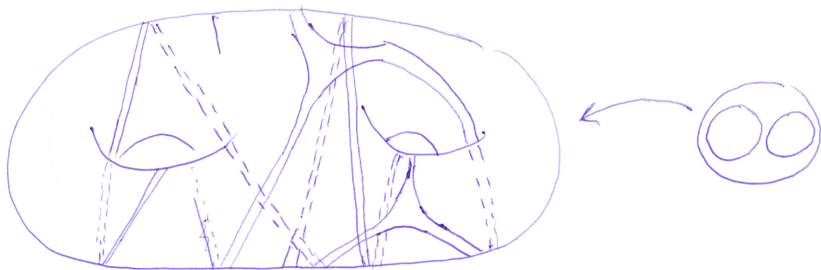[3] image from Wikipedia page "Pair of pants (mathematics)"

Pair of Pants Decomposition

Immersions of Pants

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Project Goals

- End goal: A proof of the Ehrenpreis conjecture for the cusped case.

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[6] image from https://kconrad.math.uconn.edu/blurbs/grouptheory/SL(2,Z).pdf

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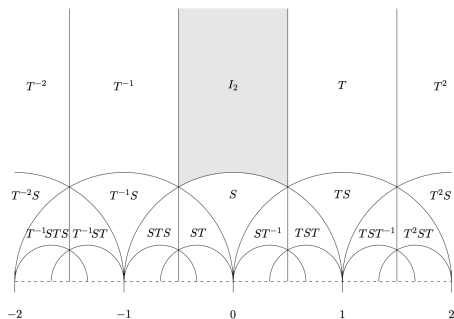
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- End goal: A proof of the Ehrenpreis conjecture for the cusped case.
- To start off, we need to understand immersions of pants and pants decompositions and examples for some model spaces.
- First space to consider: The fundamental domain of $\mathbb{H}$ over $PSL(2,\mathbb{Z}) = SL(2,\mathbb{Z}) / \{\pm I\}$. How do we put pants on it?

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