

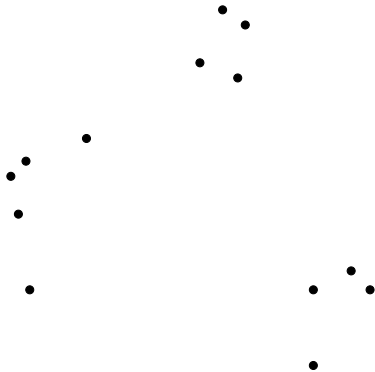
# Hardness of Approximating $k$ -Diameter

Kyrylo Karlov   Ashwin Padaki   Styopa Zharkov

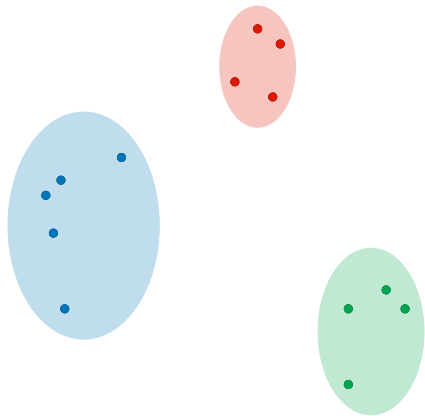
**Mentors:** Karthik C.S. and Henry Fleischmann



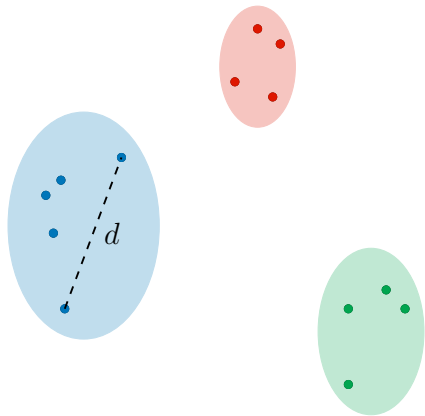
# Clustering



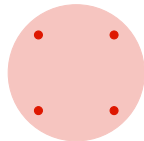
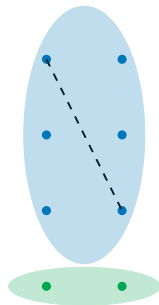
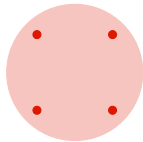
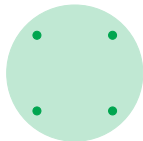
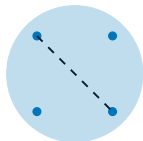
# Clustering



# Clustering



# Approximate Clustering



# Results

Metric	# of Clusters	Hardness	Best Algorithm
$l_\infty; l_0/l_1$	unbounded	2	2
$l_2$	unbounded	1.97	2
$l_\infty$	constant ( $\geq 3$ )	2	2
$l_0/l_1$	constant ( $\geq 3$ )	1.5	2
$l_2$	constant ( $\geq 3$ )	1.304	1.414

$$1.304 \approx \sqrt{1 + \sqrt{1/2}} - 0.002$$

$$1.414 \approx \sqrt{2}$$

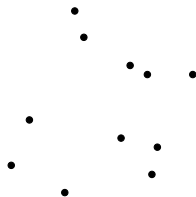
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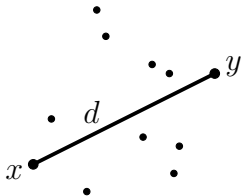
$$1.414 \approx \sqrt{2}$$

## $\ell_2$ : “Eyeball” Algorithm ( $\sqrt{2}$ -approximation)

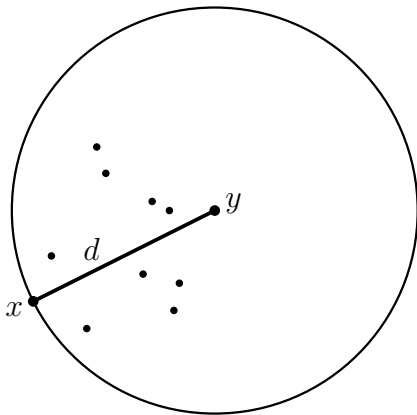




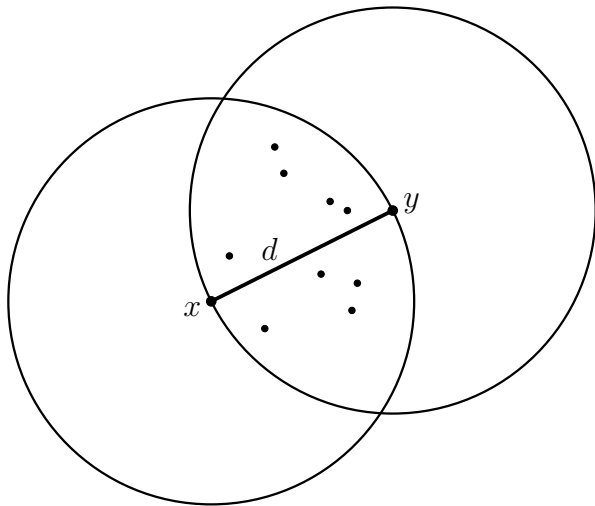
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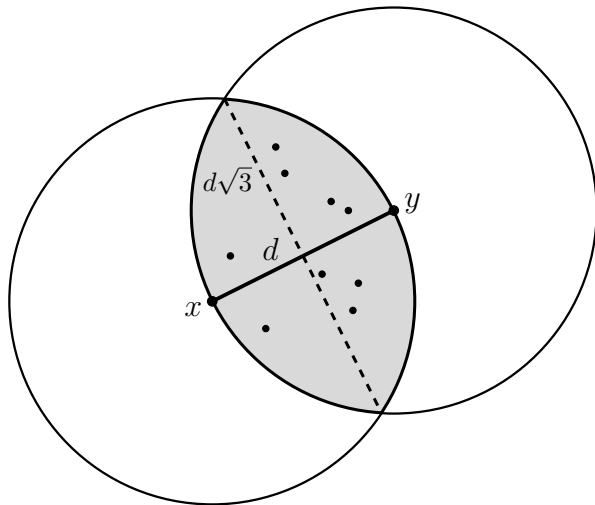
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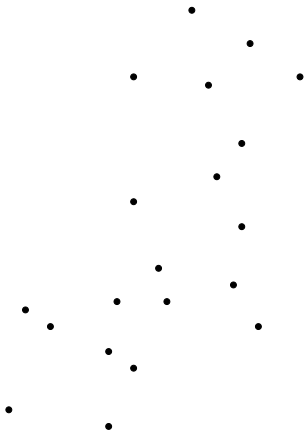
## $\ell_2$ : “Eyeball” Algorithm ( $\sqrt{2}$ -approximation)



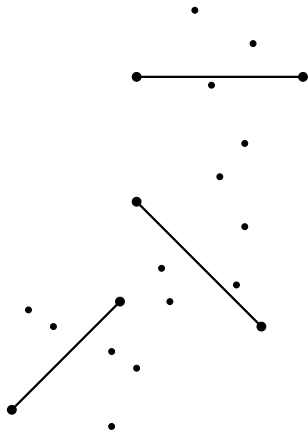
## $\ell_2$ : “Eyeball” Algorithm ( $\sqrt{2}$ -approximation)



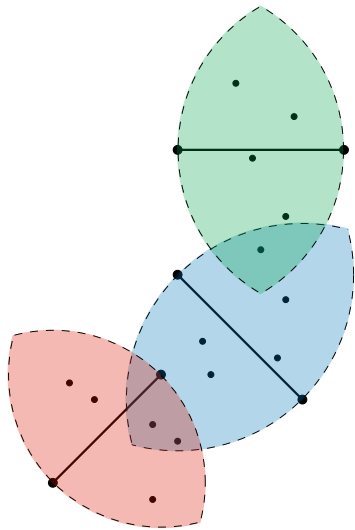
## $l_2$ : “Eyeball” Algorithm ( $\sqrt{2}$ -approximation)



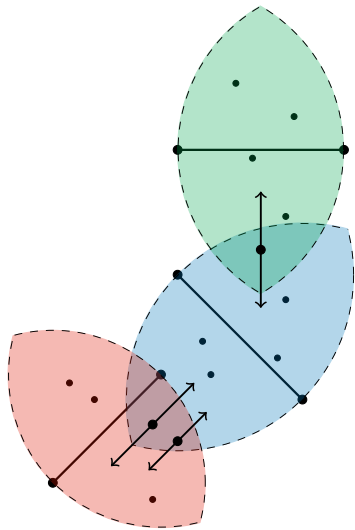
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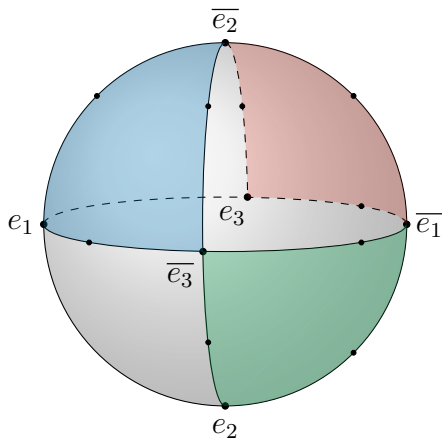
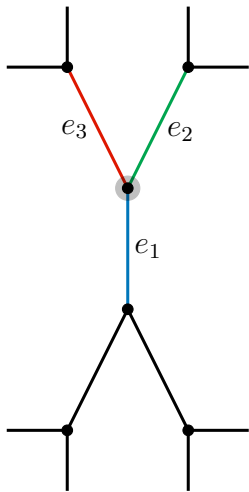
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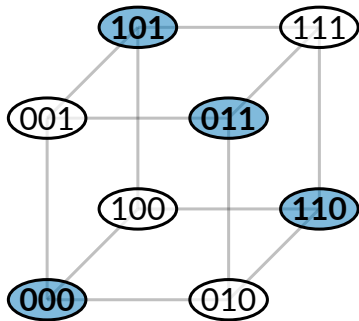
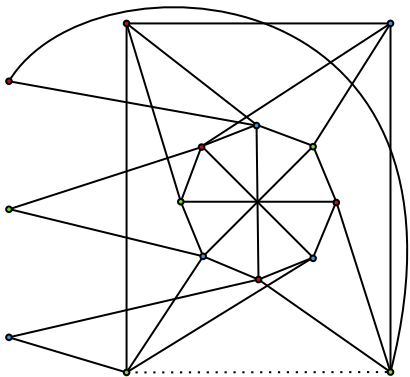
$$1.304 \approx \sqrt{1 + \sqrt{1/2}} - 0.002$$

$$1.414 \approx \sqrt{2}$$

## $\ell_2$ : Edge Coloring Reduction (1.304-hardness)



# $\ell_0$ : Vertex Coloring Reduction (1.5-hardness)





# Acknowledgements

- This work was carried out while Ashwin Padaki and Styopa Zharkov were in the 2023 DIMACS REU program at Rutgers University, supported by NSF grant CNS-2150186.
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