Synchronization in Modular Multilayer Networks

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Kayla Cummings
Pomona College

Mentors: Dr. L. Gallos, Dr. R. Wright
DIMACS, Rutgers University

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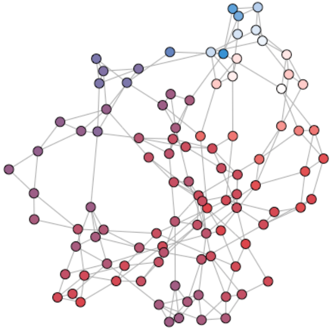
Outline



1. Kuramoto model
2. Modular networks
3. Two-layer networks
4. Future steps

Simple Kuramoto Model

Can be solved directly:

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin(\theta_j - \theta_i) \quad (i = 1, \ldots, N)$$

$$\downarrow$$

$$\frac{d\theta_i}{dt} = \omega_i + Kr\sin(\phi - \theta_i) \quad (i = 1, \ldots, N)$$

with $\quad re^{i\phi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$

Simple Kuramoto Model

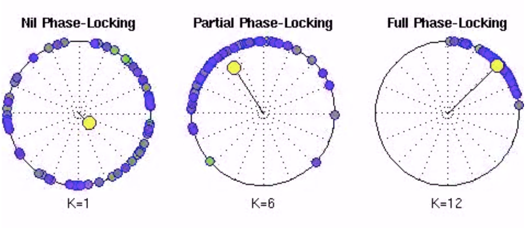


Figure 1: Visualization of Phase Locking for Different Coupling Strengths

"Kuramoto model," *Wikipedia.* Web. Accessed 2 June 2017. https://en.wikipedia.org/wiki/Kuramoto_model

Modified Kuramoto Model

Must be solved analytically:

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{\deg(i)} \sum_{j=1}^{N} a_{ij} \sin(\theta_j - \theta_i)$$

$$\downarrow$$

$$\theta_i(t+h) \leftarrow \theta_i(t) + h \cdot \left(\omega_i + \frac{K}{\deg(i)} \sum_{j=1}^{N} a_{ij} \sin(\theta_j - \theta_i)\right)$$

Modular networks

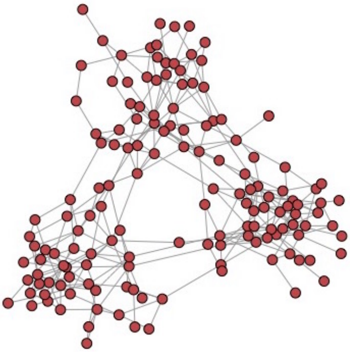


Figure 2: $\alpha = 30, m = 3, N = 150$

α: modularity strength

m: number of modules

$k_{total} = 4$: average node degree

$\downarrow$

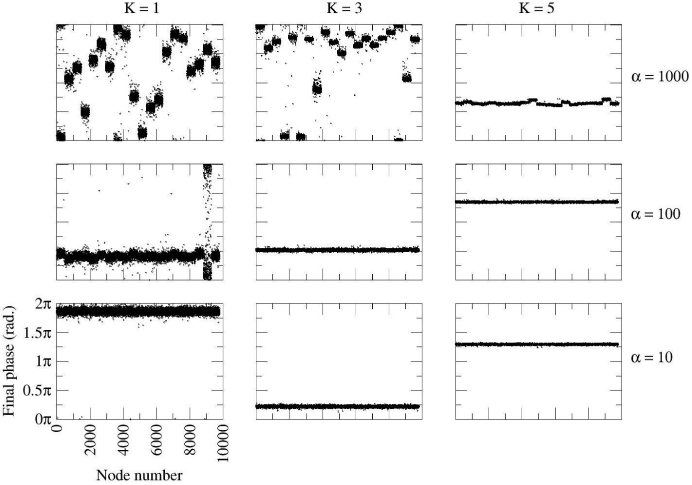
k_{inter}, k_{intra}: average intermodular and intramodular degrees

$$k_{total} = k_{inter} + k_{intra}$$

$$\frac{k_{inter}}{k_{intra}} = \frac{\alpha}{m-1}$$

Shekhtman, Shai, Havlin. *New Journal of Physics* 17.

Visualizing final phases: modularity strength ($m = 20$)



Heat maps: modularity strength

$$k_{\text{total}} = 4,\ m = 20,\ N \approx 10,000$$

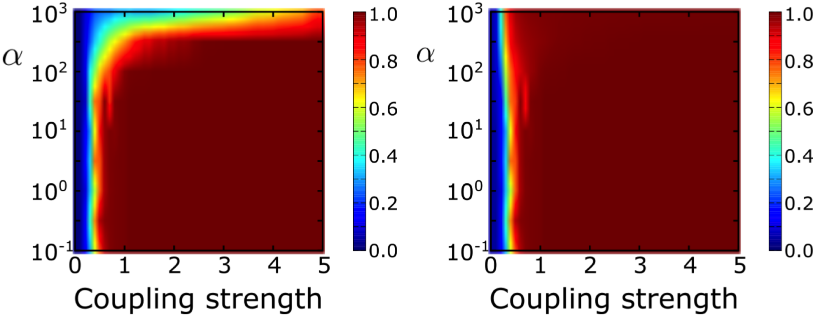


Figure 3: Left: global synchronization. Right: modular synchronization.

Heat maps: modularity strength

$$k_{\text{total}} = 4, \; m = 20, \; N \approx 10,000$$

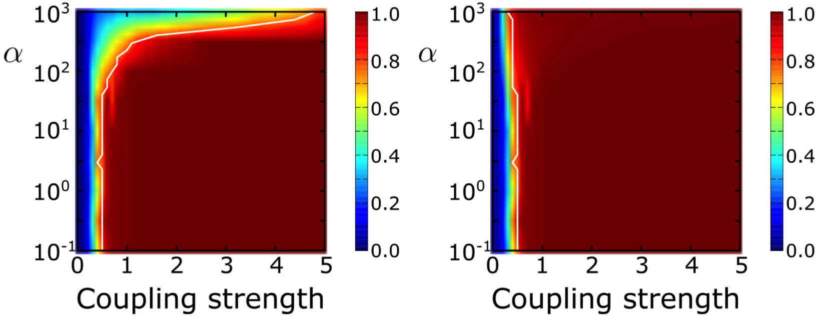
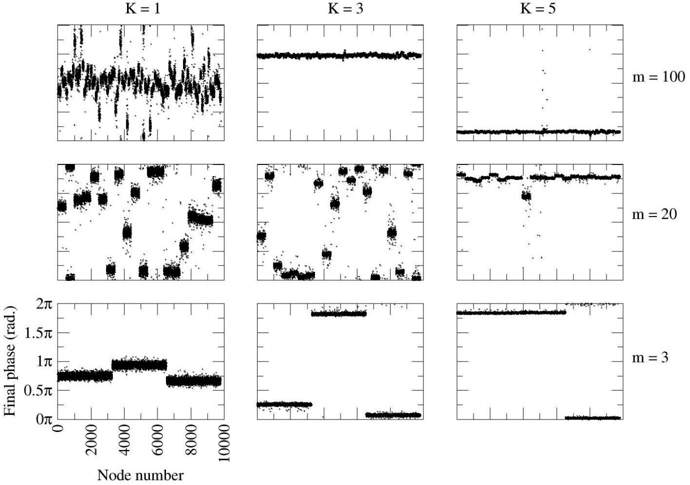


Figure 4: Left: global synchronization. Right: modular synchronization.

Visualizing final phases: number of modules ($\alpha = 1000$)



Heat maps: number of modules

$$k_{\text{total}} = 4, \alpha = 1000, N \approx 10,000$$

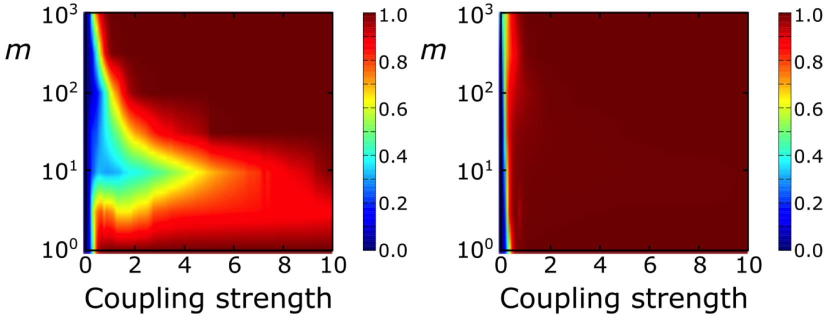


Figure 5: Left: global synchronization. Right: modular synchronization.

Heat maps: number of modules

$$k_{\text{total}} = 4, \alpha = 1000, N \approx 10,000$$

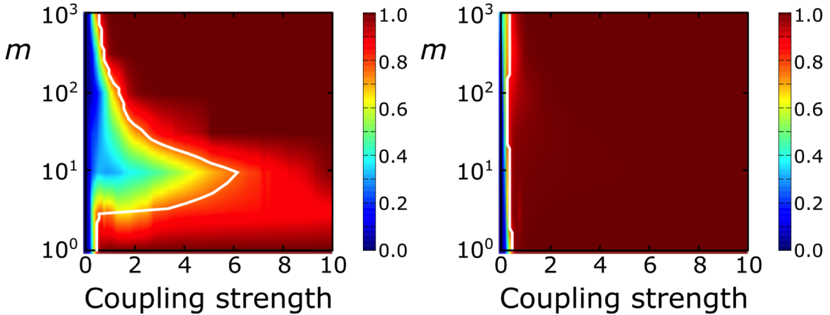
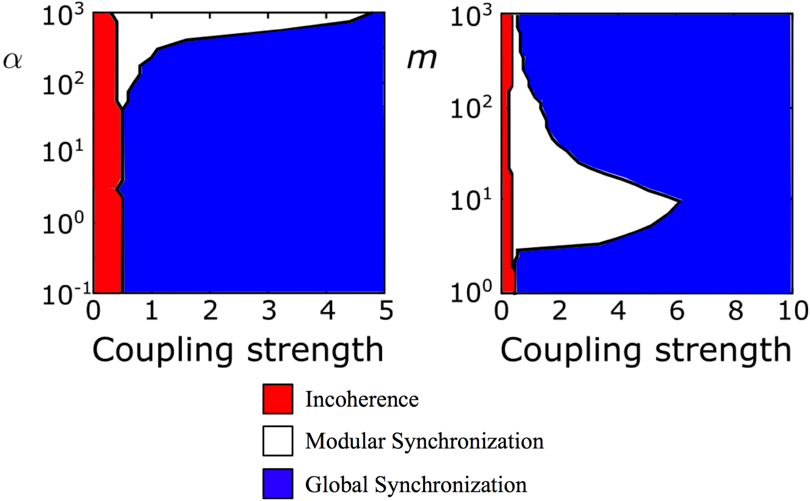


Figure 6: Left: global synchronization. Right: modular synchronization.

Phase diagram sketches: modularity and synchronization

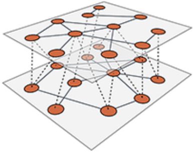


Conclusions

These phase diagrams show us that module strength and quantity can severely impair global coherence in modular networks after synchronization. However, after a relatively stable coupling strength threshold, order still exists within modules.

Two-layer modular networks

Modules $\longleftrightarrow$ Layers

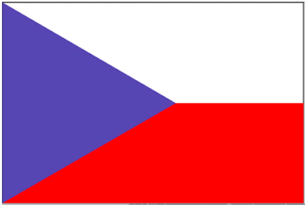


Next: heat maps showing effects of interlayer connectivity and interlayer coupling strength on global and modular synchronization in each layer and the entire network

Radicchi, F. "Driving Interconnected Networks to Supercriticality." Phys. Rev. X 4, 021014 (2014). 22 April 2014.

Next steps

1. Finalize heat maps and phase diagrams for two-layer modular networks.
2. Learn about cool combinatorial mathematics in Prague!
3. Investigate optimal modular topology for synchronization in two-layer networks.



Acknowledgements

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