Peeling in graphs

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Definition of peeling

- Peeling number of the vertex $v$ is the number $\text{peel}(v)$ defined by the following process.
  1. $p = 0$
  2. Repeat
     1. While exists vertex $v$ such that $\text{deg}(v) \leq p$
        1. $\text{peel}(v) = p$
        2. remove $v$
     2. $p = p + 1$
  3. End if no vertex remains
Properties

• Peeling numbers are unambiguous.
• Vertices of graph are divided into layers.
• These layers are sometimes called $k$-cores
• $\text{peel}(v) \leq \text{deg}(v)$ ...for every vertex $v$
• The maximal peeling number of a graph equals the degeneration of a graph.
Motivation of peeling

• Peel numbers are studied in large networks.
  – Internet network
  – Social network

• We want to determine the peel number of a given vertex online in a gradually evolving graph.
  – Our aim is to develop a data structure for this.
Data structure

• For a graph $G = (V, E)$ we want to regularly process the following queries.
  a) Initialize the structure by the graph $G$
  b) Add an edge $(u, v)$ into $G$
  c) Delete an edge $(u, v)$ from $G$
  d) Determine a peel number of a vertex $v$ at the time $t$

• We want to achieve the best amortized time complexity for queries b), c), d) and the best time complexity for a one-time query a).
Characterization

• We want to characterize graphs which vertices all have the same peeling number. Lie in the same layer.

• We determine this class of graphs as $GF_k$
  – $GF_1$ is equivalent to trees
  – $GF_2$ is equivalent to ???
  – ...
  – $GF_k$ is equivalent to ???
References
