Group-ISO

Problem proposed by professor Periklis Papakonstantinou.

Czech group

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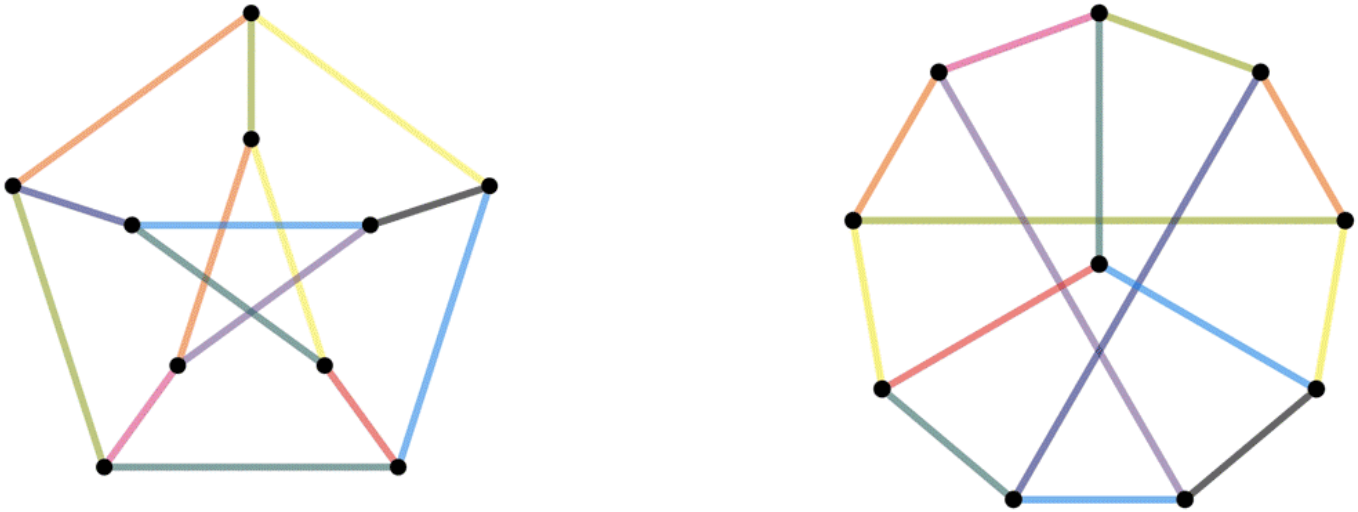
Graph isomorphism

Given two graphs decide whether their structure is the same.

http://reu.dimacs.rutgers.edu/∼svoboda/isomor.gif



Source: https://www.quantamagazine.org/20151214-graph-isomorphism-algorithm/

Graph isomorphism hardness

- Graph isomorphism is believed not to be hard (not NP complete).
- It may or may not be polynomial (in P).
- It might be something in between (which would be cool!).

Group isomorphism

Group is an abstract algebraic structure with a binary operation.
An example is $\mathbb{Z}$ with addition or $\mathbb{R} \setminus \{0\}$ with multiplication.

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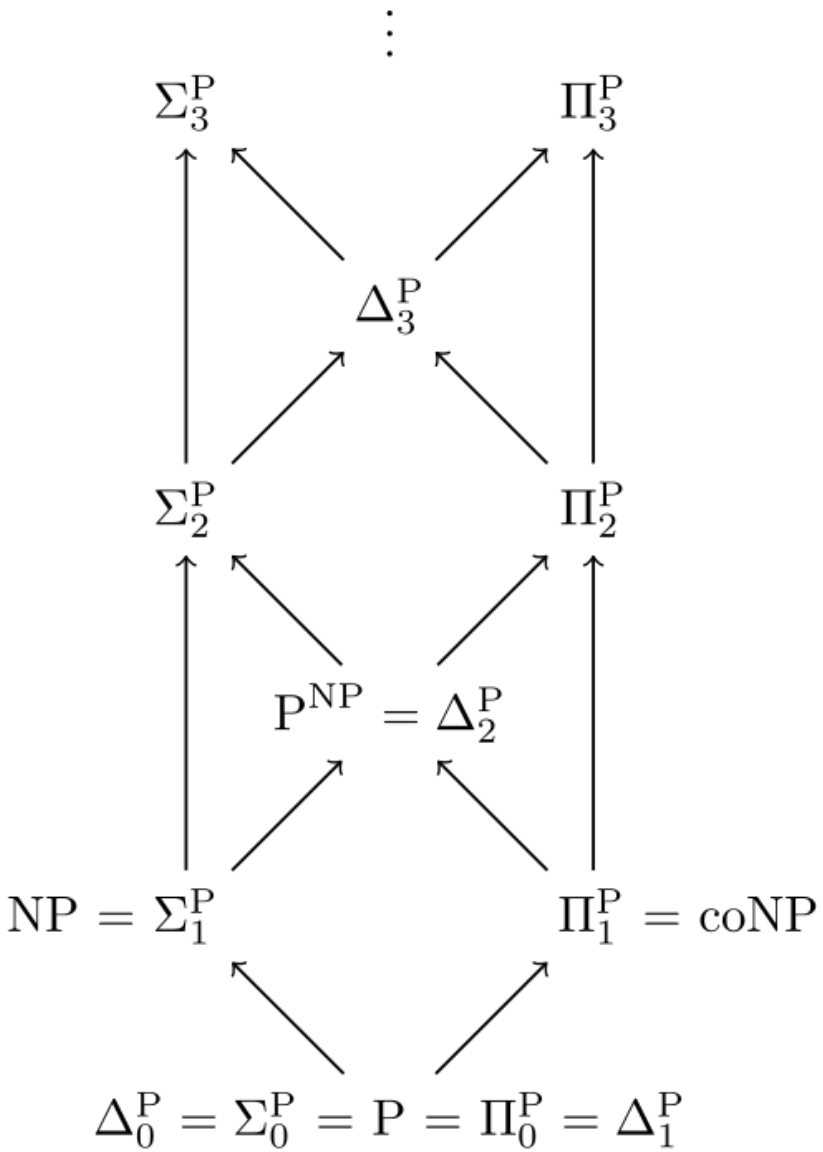
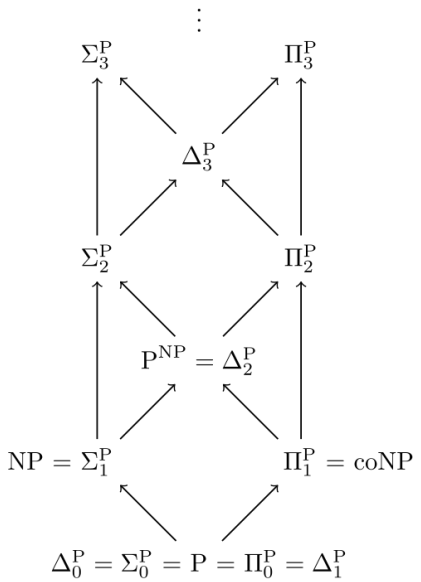
Given two finite groups decide whether their structure is the same.

This problem is easier than graph isomorphism because we can create a graph from the group and then test isomorphism there.

Polynomial hierarchy



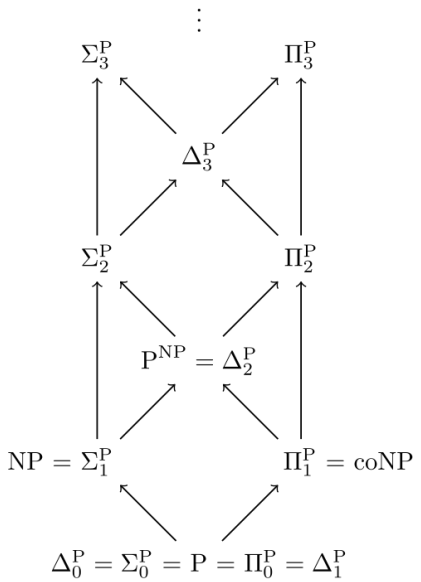
Polynomial hierarchy



Theorem

If graph isomorphism is *NP*-complete then
$\Sigma_2^P = \Sigma_3^P = \cdots = \Pi_2^P = \Pi_3^P = \ldots$

Polynomial hierarchy



Theorem

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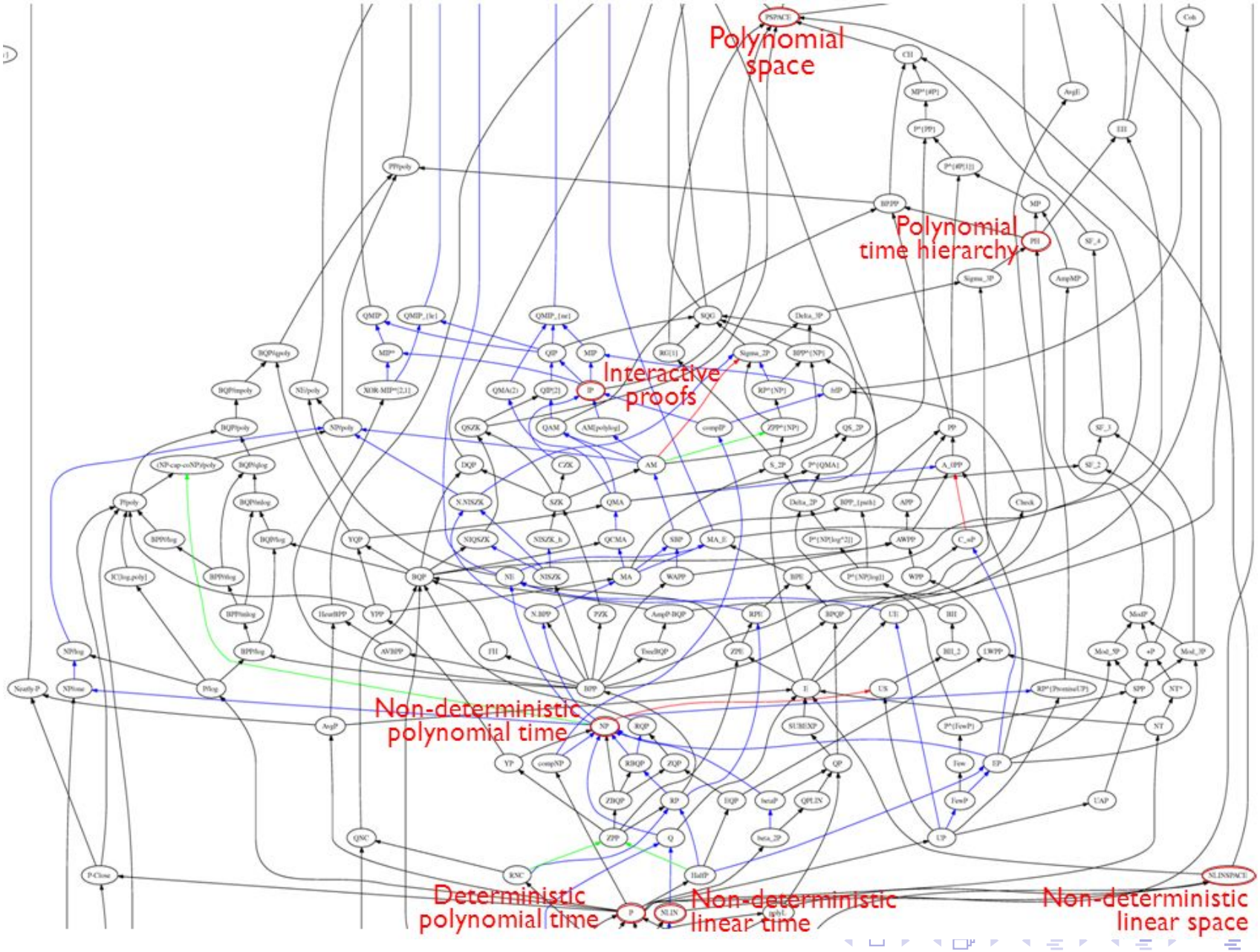
This basically means that if graph isomorphism is hard then polynomial hierarchy collapses.

Our aim

We want to prove an analogous theorem for group isomorphism showing that this problem is probably even closer to P (otherwise some other hierarchy collapses).

What they did not tell you about complexity classes



Acknowledgements

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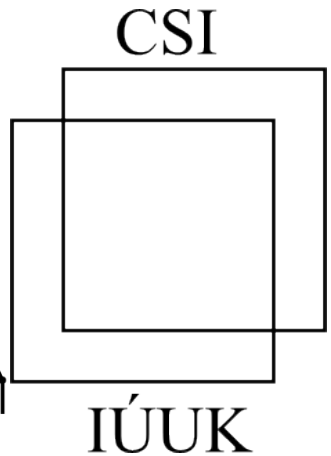

Questions

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