A Game Theory Approach to Cascading Behavior in Networks

By
Jim Manning
Jordan Mitchell
Ajay Mattappallil
What is Viral Marketing?

• Start off with a small group of individuals
• “Word-of-Mouth”
• Initial individual tells two of his friends and they tell...
• Not necessarily selling a product
• Viral ➔ Think Virus
Small-Scale Behavioral Viral Spreading

• Two people start singing

• Next their friends join in

• By the end of the song, the whole bar is singing Mariah Carey’s Fantasy...true story
Graph Theory Basics

Nodes

Directed & Undirected

Edges

Directed & Undirected

Diagram of a graph with nodes and edges.
Graph Theory relating back to VM

- Contagious
- Progressive & Nonprogressive
- Weighted edges
- “Target” node
Game Theory

- Prisoners’ Dilemma
- Two choices
- Confess
- Don’t Confess
• Payoff
• Prisoners’ Outcomes
• Payoff Table
- Zero Sum and Nonconstant Sum Games
- Nash Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>Confess</th>
<th>Don’t Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>(10,10)</td>
<td>(0,20)</td>
</tr>
<tr>
<td>Don’t Confess</td>
<td>(20,0)</td>
<td>(1,1)</td>
</tr>
</tbody>
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- Clean Kitchen: $50, Clean Basement: $30, Clean Living Room: $20

<table>
<thead>
<tr>
<th></th>
<th>Kitchen</th>
<th>Basement</th>
<th>Living Room</th>
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<tr>
<td>Kitchen</td>
<td>(25,25)</td>
<td>(50,30)</td>
<td>(50,20)</td>
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<tr>
<td>Basement</td>
<td>(30,50)</td>
<td>(15,15)</td>
<td>(30,20)</td>
</tr>
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• Apply Game Theory to Viral Marketing

• Who are the players?

• Behaviors
Agent i’s Payoff

- \( v_i \ g(d_i) \ \pi_i - c_i \)
  - \( v_i \) = benefit from switching
  - \( g(d_i) \) = how number of neighbors affects switch
  - \( \pi_i \) = % of neighbors already switched
  - \( c_i \) = cost of switching

- Will switch if \( (v_i / c_i) \ g(d_i) \ \pi_i > 1 \)

- Let \( F \) be cdf of \( v_i / c_i \)
Number of Neighbors Effect

- One possible $g(d) = ad^\beta$

- If $\beta = 0$: Only fraction of neighbors activated matters. Matched games.
- If $\beta = 1$: $g(d_i) \pi_i$ is proportional to number of neighbors activated. Epidemiology.
Determining Percent Activated

- \( P(d) \) is the connectivity distribution
  - i.e. percentage of agents in population with exactly \( d \) direct neighbors
- \( x^t \) = percentage activated at time \( t \)

\[
x^t = 1 - \frac{1}{d} \sum_d d \ P(d) \ F\left[ \frac{1}{g(d) x^{t-1}} \right]
\]
Connectivity Distribution

- Three example types:
  - Scale-free networks (power law)
    \[ P(k) \propto k^{-\gamma}, \quad 2 < \gamma < 3 \]
  - Homogenous Networks
    \[ P(k) \sim \begin{cases} 
      0, & k \neq \bar{k} \\ 
      1, & k = \bar{k} 
    \end{cases} \]
  - Poisson Networks
    \[ P(k) = \frac{1}{k!} e^{-\bar{k}} \bar{k}^k \]
- Larger variance => Product spreads better
Tipping Point and Equilibrium

![Graph showing tipping point and equilibrium]

- Adoption at $t+1$
- Adding connections
- Tipping point in original network
- Stable equilibrium moves up
Most Influential Nodes

- In general, identifying $k$ most influential nodes is NP-hard.
- A natural greedy algorithm exists which is a $1-1/e-\varepsilon$ approximation for selecting a target set of size $k$, using probabilistically determined simulations.
Our Research Direction

- Looking for graph types which are not susceptible to strong inapproximability results, and for which good approximation results can be obtained.
- Consider graphs with edges weighted differently.
- Analyze real world data from social networks or viral marketing campaigns.
Sources

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Diffusion in Complex Social Networks (Dunia Lopez-Pintado):

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Cascading Behavior in Networks: Algorithmic and Economic Issues (Jon Kleinberg):
Thank you for your attention!

Questions?