

Understanding non-monotonicity for 2 independent items

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Problem Definition

Two item sale

- bids: $v \sim D$,
- allocation: $a : D \times D \rightarrow [0, 1]^2$,
- price: $p : D \times D \rightarrow \mathbb{R}$.

Seller's profit

$$Rev(D) = \sum_{\vec{v} \in \text{supp}(D \times D)} d(\vec{v})p(\vec{v}).$$

Stochastic Dominance

$$D \leq D' \Leftrightarrow \forall x \in \mathbb{R}, v \sim D, v' \sim D' : Pr(v > x) \leq Pr(v' > x).$$

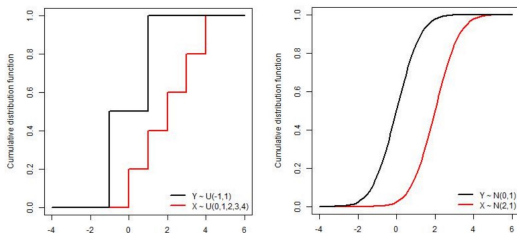


Figure: Example of stochastic dominance.

Weird Example

$D \leq D'$, but $Rev(D) > Rev(D')$!

$$D = \begin{cases} 10 & \text{wp } \frac{4}{15} \\ 46 & \text{wp } \frac{1}{90} \\ 47 & \text{wp } \frac{1}{3} \\ 80 & \text{wp } \frac{7}{30} \\ 100 & \text{wp } \frac{7}{45} \end{cases}$$

$$D' = \begin{cases} 10 & \text{wp } \frac{2399}{9000} \\ 13 & \text{wp } \frac{1}{9000} \\ 46 & \text{wp } \frac{1}{90} \\ 47 & \text{wp } \frac{1}{3} \\ 80 & \text{wp } \frac{7}{30} \\ 100 & \text{wp } \frac{7}{45} \end{cases}$$

Research Questions

- Understand weird example.
- Construct more examples.
- Are these examples rare?
- Do they concentrate around certain distributions?