Concordance Invariants of Satellite Knots

J. Patwardhan[1] Z. Xiao[2]

[1]Rutgers University

[2]Columbia University

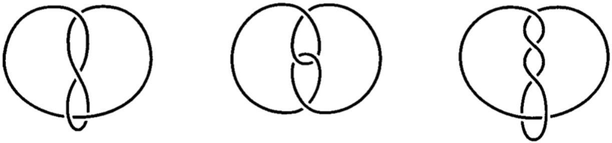
DIMACS REU

What is a knot?

A **knot** is a loop of string in $\mathbb{R}^3$, which has no thickness, with its cross-section being a single point. (Formally, we say a knot is an embedding $S^1 \hookrightarrow S^3$.)

(a) The unknot. (b) A trefoil knot.

There are many different pictures of the same knot. Below are all pictures of the figure eight knot.



Why study knots

We study knots because they are closely related to 3 and 4 dimensional manifolds.

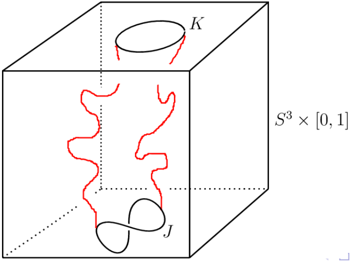
Theorem (Lickorish, Wallace, 1960s)

Every closed orientable 3-dimensional manifold can be described in terms of a collection of knots and an integer associated to each knot.

Note that 3D manifolds are hard to visualize, but knots are not!

Concordance

- In general, knots are often studied up to a notion of equivalence, called knot concordance.
- Two knots are said to be **concordant** if they jointly form the boundary of a cylinder in S^3.
- Formally speaking, two knots K and J are said to be **concordant** ($K \sim J$) if there is an embedding $f : S^1 \times [0, 1] \to S^3 \times [0, 1]$ such that $f(S^1 \times 0) = K$ and $f(S^1 \times 1) = J$
- The set of concordance classes of knots form a group, denoted $\mathscr{C}$.

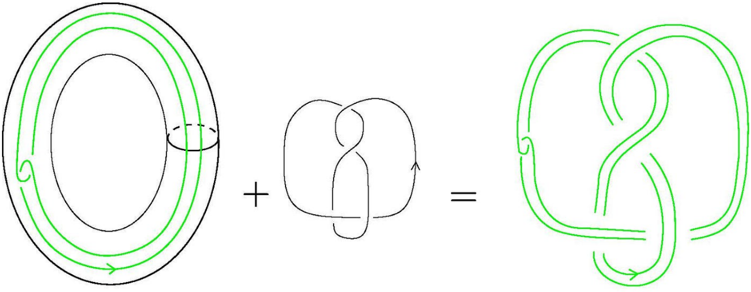


τ-invariant

- In 2003, P. Ozsváth and Z. Szabó defined an invariant of the concordance class of a knot, called the τ-invariant.
- Formally, the τ-invariant is a group homomorphism $\tau : \mathscr{C} \to \mathbb{Z}$ which sends all elements of a concordance class to an integer. It allows us to distinguish which knots are not concordant to each other.
- The goal of our project is to compute τ for specific types of knots, called satellite knots.

Satellite Knot

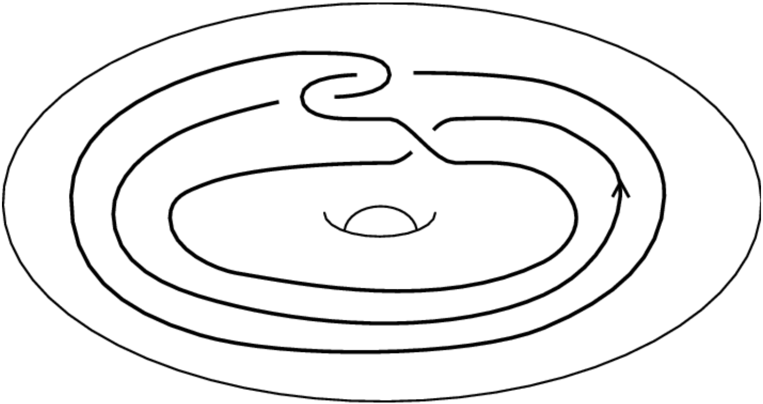
- A fundamental operation of producing new knots out of existing ones is called **the satellite operation.**
- A satellite knot has two components: a pattern knot P (embedded in a solid torus) and a companion knot K. Cut up the torus and glue it back according to K. The image of P under this process is called the **satellite knot with pattern P and companion K**. Denote it by $P(K)$.
- E.g., let P be the Whitehead double, K be the figure eight



$P \subset S^1 \times D^2$ $\quad$ $K \subset S^3$ $\quad$ $P(K) \subset S^3$

Mazur Pattern

We are interested in the satellite knots coming from the Mazur pattern, shown below, as well as generalizations of this pattern.



In 2016, A. Levine used a knot invariant called **bordered Floer homology** which is well-adapted to studying the satellite construction to give a formula of calculating the tau-invariant of satellite knots with Mazur patterns.

Theorem (Levine, 2016)

Let Q denote the Mazur pattern shown in Figure 2. For any knot $K \subset S^3$,

$$\tau(Q(K)) = \begin{cases} \tau(K) & \text{if } \tau(K) \leq 0 \text{ and } \epsilon(K) \in \{0, 1\}, \\ \tau(K) + 1 & \text{if } \tau(K) > 0 \text{ or } \epsilon(K) = -1. \end{cases}$$

We will be using this process to compute tau-invariants for general Mazur patterns.

Acknowledgements

We would like to thank

- Professor Hendricks and Professor Mallick
- Rutgers Department of Mathematics
- NSF CAREER Grant DMS-2019396

References

- Adam Levine, "Non-surjective satellite operators and piecewise-linear concordance", Forum of Mathematics (2014), Sigma. 4. 10.1017/fms.2016.31.
- Collin Adams, *Knot Book* (1994), W.H. Freeman.
- Dale Rolfsen, *Knots and links*, American Mathematical Society (2004).
- Peter Ozsváth and and Zoltán Szabó, "Knot Floer homology and the four-ball genus", Geom. Topol. 7 (2003), 615-639.
- Ruth A. Situma, "One Knot at a Time: PIMS CRG PDF Wenzhao Chen, on Knot Theory and Classification.," `https://medium.com/pims-math/one-knot-at-a-time-pims-crg-pdf-wenzhaochen-\on-knot-theory-and-classification-355a3d4dd7fb`.

Thanks for your time!