FAIR ALLOCATION WITH INDIVISIBLE BUT SHAREABLE GOODS

By: Martin Černý Hana Salavcová



FAIR ALLOCATION PROBLEM

FAIR ALLOCATION PROBLEM: I = (N

 $\blacksquare N = \{1, \ldots, n\} \dots \text{ agents}$



FAIR ALLOCATION PROBLEM: I = (N, M)

■
$$N = \{1, ..., n\}$$
 ... agents ■ $M = \{1, ..., m\}$... goods





FAIR ALLOCATION PROBLEM: I = (N, M, v)

■ $N = \{1, ..., n\}$... agents ■ $M = \{1, ..., m\}$... goods ■ $v = (v_1, ..., v_n)$... valuations

$$v_i(S) = \sum_{g \in S} v_i(\{g\}), \quad \forall S \subseteq M$$





•
$$A = (A_1, \ldots, A_n)$$
 ... allocations



$$\blacksquare A = (A_1, \dots, A_n) \dots \text{ allocations}$$

1. $A_i \subseteq M$



■
$$A = (A_1, ..., A_n)$$
 ... allocations
1. $A_i \subseteq M$
2. $A_i \cap A_j = \emptyset$



■
$$A = (A_1, ..., A_n)$$
 ... allocations
1. $A_i \subseteq M$
2. $A_i \cap A_j = \emptyset$
3. $\bigcup A_i = M$



■
$$A = (A_1, ..., A_n)$$
 ... allocations
1. $A_i \subseteq M$
2. $A_i \cap A_j = \emptyset$
3. $\bigcup A_i = M$















 $v_i(A_i) \geq v_i(A_i)$



Allocation $A = (A_1, ..., A_n)$ for (N, M, v) is envy-free if for $i, j \in N$:

 $v_i(A_i) \geq v_i(A_j)$









SOLUTION CONCEPTS: OVERVIEW













•
$$A = (A_1, \ldots, A_n)$$
 ... allocations



•
$$A = (A_1, \dots, A_n)$$
 ... allocations
1. $A_i \cap A_j \cap A_k = \emptyset$



•
$$A = (A_1, \dots, A_n)$$
 ... allocations
1. $A_i \cap A_j \cap A_k = \emptyset$
2. $\bigcup A_i = M$



COST OF SHARING



COST OF SHARING







We would like to gratefully acknowledge the guidance of our supervisor **Arpita Biswas** throughout the course of this project.

We would also like to thank the **DIMACS REU 2025 program** for providing us with this incredible research opportunity. We are especially grateful to **Rutgers University** and the **DIMACS center** for hosting the program and creating a stimulating and supportive research environment.

The authors, Martin Černý and Hana Salavcová, are further partly supported by

- Department of Applied Mathematics,
- Informatics Institute of Charles University,
- RSJ foundation.