Hardness of Approximating *k*-clique

Reina Itakura, Mayank Motwani, Gary Peng

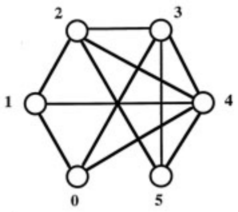
Mentor: Prof. Karthik Srikanta

DIMACS, 6.3.25

The k-clique Problem

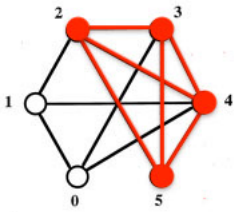
The Problem:

- Input: Graph $G = (V, E)$, integer k
- Output: Find k-clique in G.



The Problem:

- Input: Graph $G = (V, E)$, integer k
- Output: Find k-clique in G.



Bad News

- k-clique is NP-complete (i.e., poly(n)-time algorithm unlikely)
- k-clique is W[1]-complete (i.e., $f(k)$poly(n)-time algorithm unlikely)
- no $n^{o(k)}$-time algorithm under ETH
- believed to have no algorithm better than $n^{\omega k/3}$-time

Approximating k-clique

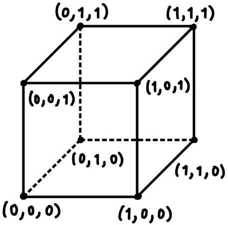
1. trivial: k approximation
2. NP-Hard to approximate to $n^{1-\epsilon}$ factor for any $\epsilon > 0$ (Håstad'96)
3. Assuming Gap-ETH, no $o(k)$-approx. in FPT time (Chalermsook et al.'17)
4. W[1]-complete to approximate to constant factor (Lin'21)
5. W[1]-complete to approximate to $k^{o(1)}$ factor (Karthik-Khot'21)
6. Assuming ETH, no $o(k/\text{polylog}(k))$-approx. in FPT time (Bafna-Karthik-Minzer'25)
7. Nothing is known in fine-grained world for explicit k.

Motivating Direction

- **Current**: use CSPs to study (in)approximability of k-clique
- **Want**: obtain results from studying k-clique directly

Current Directions

1. reducing $(i, i+1)$-gap clique to $(i, i+2)$-gap clique for small fixed i
2. using locally-decodable codes to improve self-reductions
3. PCP Theorem proof for cliques via hypercubes



Acknowledgements

We would like to thank our advisor Karthik Srikanta, his graduate student Mursalin Habib, and DIMACS REU for supporting our research.

We would also like to thank the NSF for funding our research through grants CCF-2447342, CCF-2313372, and CCF-2443697.

References

Parinya Chalermsook et al. "From Gap-ETH to FPT-Inapproximability: Clique, Dominating Set, and More". In: *2017 IEEE 58th Annual Symposium on Foundations of Computer Science (FOCS)*. 2017, pp. 743–754. DOI: 10.1109/FOCS.2017.74.

Yijia Chen et al. "Simple Combinatorial Construction of the ¡italic¿k¡/italic¿¡sup¿¡italic¿o¡/italic¿ (1)¡/sup¿-Lower Bound for Approximating the Parameterized ¡italic¿k¡/italic¿-Clique". In: *2025 Symposium on Simplicity in Algorithms (SOSA)*, pp. 263–280. DOI: 10.1137/1.9781611978315.21. URL: https://epubs.siam.org/doi/abs/10.1137/1.9781611978315.21.

Irit Dinur. "The PCP theorem by gap amplification". In: *J. ACM* 54.3 (June 2007), 12–es. ISSN: 0004-5411. DOI: 10.1145/1236457.1236459. URL: https://doi.org/10.1145/1236457.1236459.

R.G. Downey and M.R. Fellows. *Fundamentals of Parameterized Complexity*. Texts in Computer Science. Springer London, 2013. ISBN: 9781447155591.

J. Hastad. "Clique is hard to approximate within n/sup 1-/spl epsiv//". In: *Proceedings of 37th Conference on Foundations of Computer Science*. 1996, pp. 627–636. DOI: 10.1109/SFCS.1996.548522.

C. S. Karthik and Subhash Khot. "Almost polynomial factor inapproximability for parameterized k-clique". In: *Proceedings of the 37th Computational Complexity Conference*. CCC '22. Philadelphia, Pennsylvania: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2022. ISBN: 9783959772419. DOI: 10.4230/LIPIcs.CCC.2022.6. URL: https://doi.org/10.4230/LIPIcs.CCC.2022.6.

Bingkai Lin. "Constant approximating k-clique is w[1]-hard". In: *Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing*. STOC 2021. Virtual, Italy: Association for Computing Machinery, 2021, pp. 1749–1756. ISBN: 9781450380539. DOI: 10.1145/3406325.3451016. URL: https://doi.org/10.1145/3406325.3451016.