Steiner Trees for Regular Simplexes

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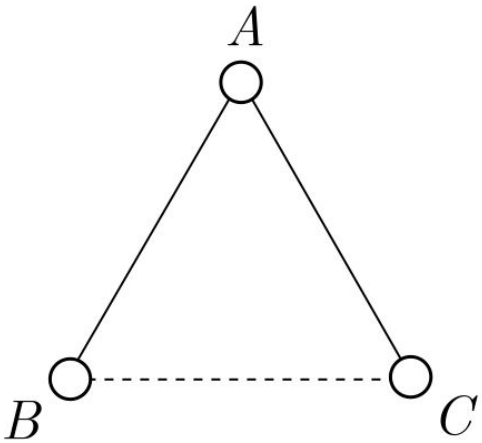
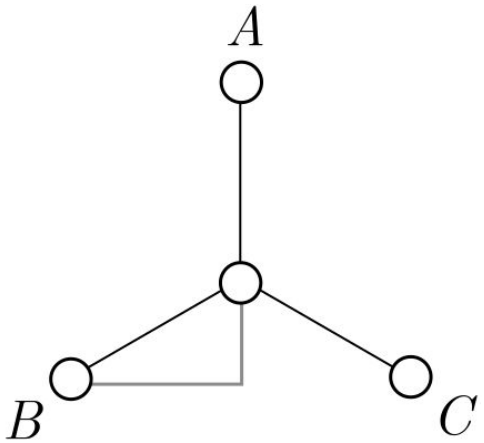
Background

- Steiner tree problem
  - Given n points find the tree that connects them while minimizing the length of the tree.
  - We may add new points - those are called Steiner points.

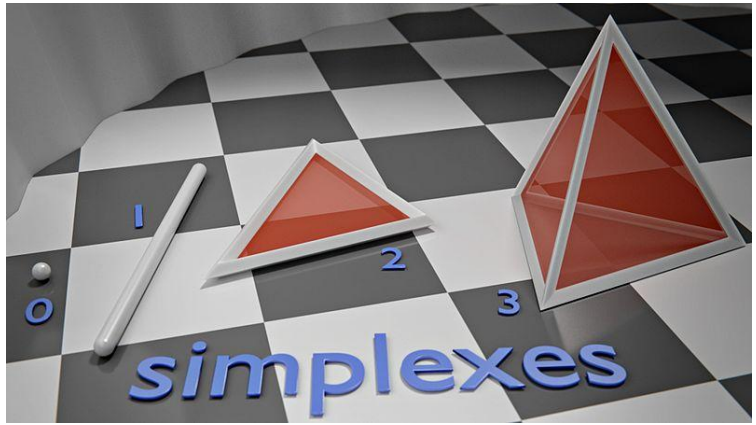




Pictures taken from https://hapax.github.io/assets/2020-03-11-steiner/

Background

- Regular simplex
  - Generalization of triangle.
  - For dimension D we have D + 1 affinely independent points
  - Take a convex hull of those points and you get a simplex.
  - If all edges has the same length then it is regular.



Picture taken from https://en.wikipedia.org/wiki/File:Simplexes.jpg

Main goal

- Improve on the results by Chung and Gilbert (1976)
  - L_S - length of the minimal Steiner Tree
  - L_M - length of the minimal tree
  - Numbers in the table are optimal for D ≤ 5
- Many possible topologies (Gilbert & Pollak, 1968)

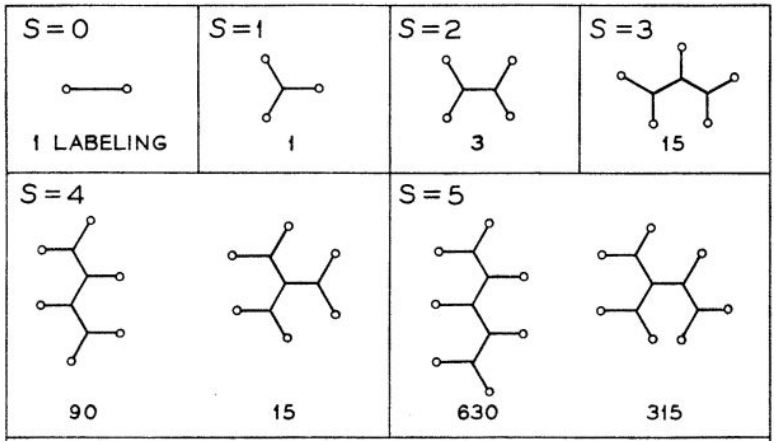
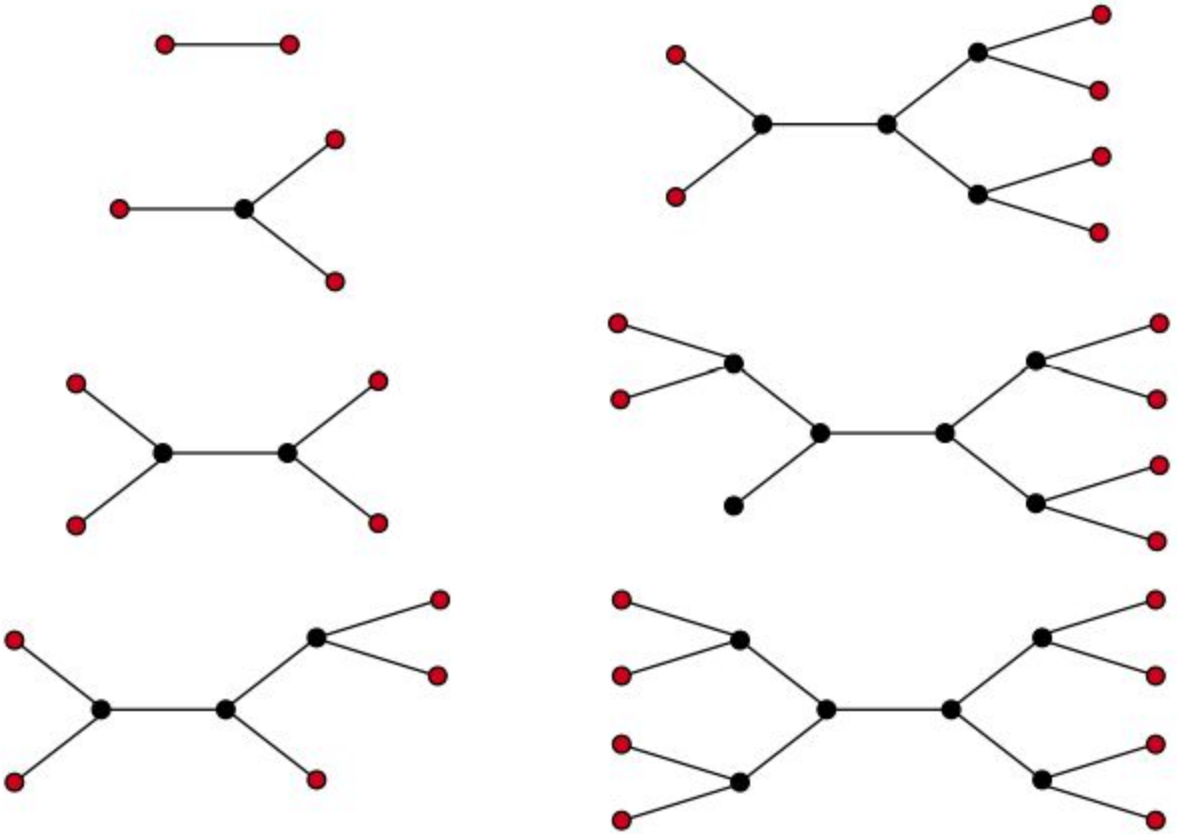


TABLE I. Upper Bound on $\rho(D) = L_S/L_M$ for a Simplex in Dimension D.

D	Bound
1	1.
2	.866026
3	.813053
4	.783748
5	.764564
6	.751427
7	.741264
8	.733982
9	.727434
10	.722504
11	.718118
12	.714967
13	.711555
14	.711033
15	.706485
16	.704923
17	.702721
18	.701083
19	.699453
20	.698390
40	.684995
80	.677754
160	.673921
large	$C = .669842$

2 Conjectured topology



Methodology: Two main sources

Gilbert, E. N., and Pollak, H. O. “Steiner Minimal Trees.” SIAM Journal on Applied Mathematics, vol. 16, no. 1, 1968, pp. 1–29. JSTOR, http://www.jstor.org/stable/2099400.

- Considers Steiner trees for points in the plane.
- Properties of Steiner trees in the plane and generalizations.
- IMPORTANT PROPERTY: In any Euclidean space, at most three lines can meet at angles greater than 120°.

Chung, F. R. K., and Gilbert, E. N. "Steiner trees for the regular simplex." Bull. Inst. Math. Acad. Sinica 4.2 (1976): 313-325.

- Consider Steiner trees for regular simplexes in arbitrary dimensions.
- Present construction of Steiner minimal trees for dimensions 3, 4 and 5.

Acknowledgement

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