What is a Mixed Discriminant?

To define the mixed discriminant, let $\vec{A} = (A_1, \ldots, A_d)$ be a collection of $d \times d$ matrices, and consider the function

$$
det_{\vec{A}} : \mathbb{R}^d \rightarrow \mathbb{R}, \quad (t_1, \ldots, t_d) \mapsto -\det(t_1A_1 + \ldots + t_dA_d)
$$

which is a homogeneous polynomial of degree $d$. The number

$$
D(A_1, \ldots, A_d) = \frac{\partial^d}{\partial t_1 \ldots \partial t_d} det_{\vec{A}}(t_1, \ldots, t_d)
$$

is called the mixed discriminant of $\vec{A}$. The mixed discriminant is symmetric in $\vec{A}$, and it is nonnegative whenever all the matrices in $\vec{A}$ are positive semidefinite.
Example

Let $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$det(xA_1 + yA_2 + zA_3) = \begin{vmatrix} x + y + z & 0 & 0 \\ 0 & x + y + z & 0 \\ 0 & 0 & x + y + z \end{vmatrix}$

$D(A_1, A_2, A_3) = \frac{\partial^3}{\partial x \partial y \partial z} (x + y + z)^3 = 6$
Let $\vec{P} = (P_1, \ldots, P_{d-2})$ be a collection of $d \times d$ positive semidefinite matrices.

$$HR(\vec{P}) : \text{Sym}_d \times \text{Sym}_d \to \mathbb{R}, \quad (\eta_1, \eta_2) \mapsto D(\eta_1, \eta_2, P_1, \ldots, P_{d-2})$$
HR(\vec{P}) can be represented by a matrix $T$.

$$T = [D(\eta_i, \eta_j, P_1, \ldots, P_{d-2})]$$
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**Theorem 1**

Any matrix representing HR(\vec{P}) has exactly one positive eigenvalue.
Goals

Long term: To show how the general theorem of matroids can be applied in various different cases, and to have a few examples to show Rota’s unimodality conjecture: If $w_k(M)$ is the number of rank $k$ flats of a rank $d$ matroid $M$, then the sequence $w_0(M), ..., w_d(M)$ is unimodal. Welsh conjectured more generally that the sequence is log-concave.

Right now: Prove that the determinant of a matrix representing one case of $HR(\vec{P})$ is nonpositive.
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