# Manifolds and complex structure

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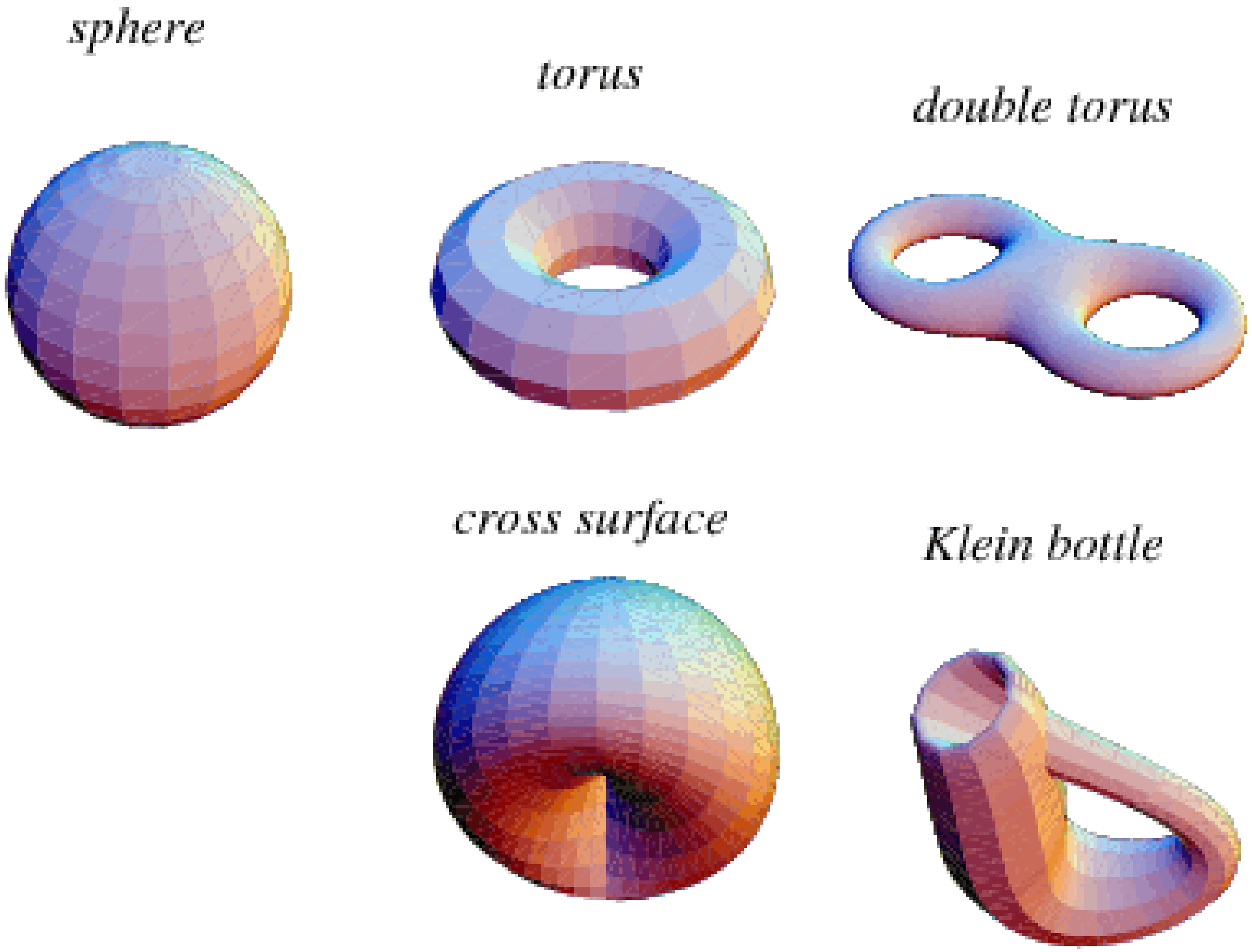
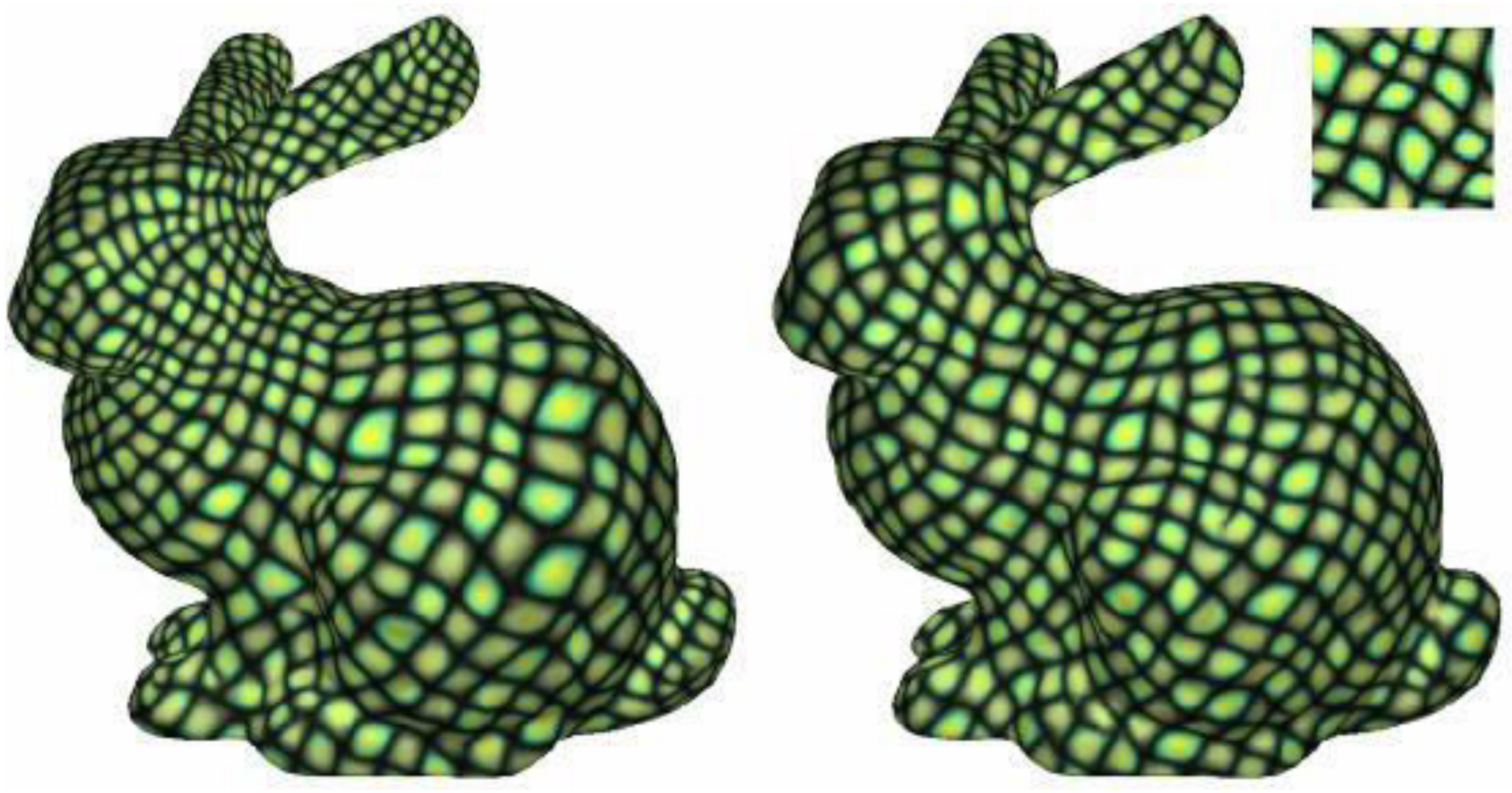
# Topological Manifolds

- An n-dimensional manifold *M* is a space that is locally Euclidean, i.e. near any point *p* of *M,* we can define points of *M* by n real coordinates
- Curves are 1-dimensional manifolds (1-manifolds)
- Surfaces are 2-manifolds
- A chart *(U, f)* is an open subset *U* of *M* and a homeomorphism *f* from *U* to some open subset of *R*
- An atlas is a set of charts that cover *M*

# Some important 2-manifolds



# A very important 2-manifold



# Smooth manifolds

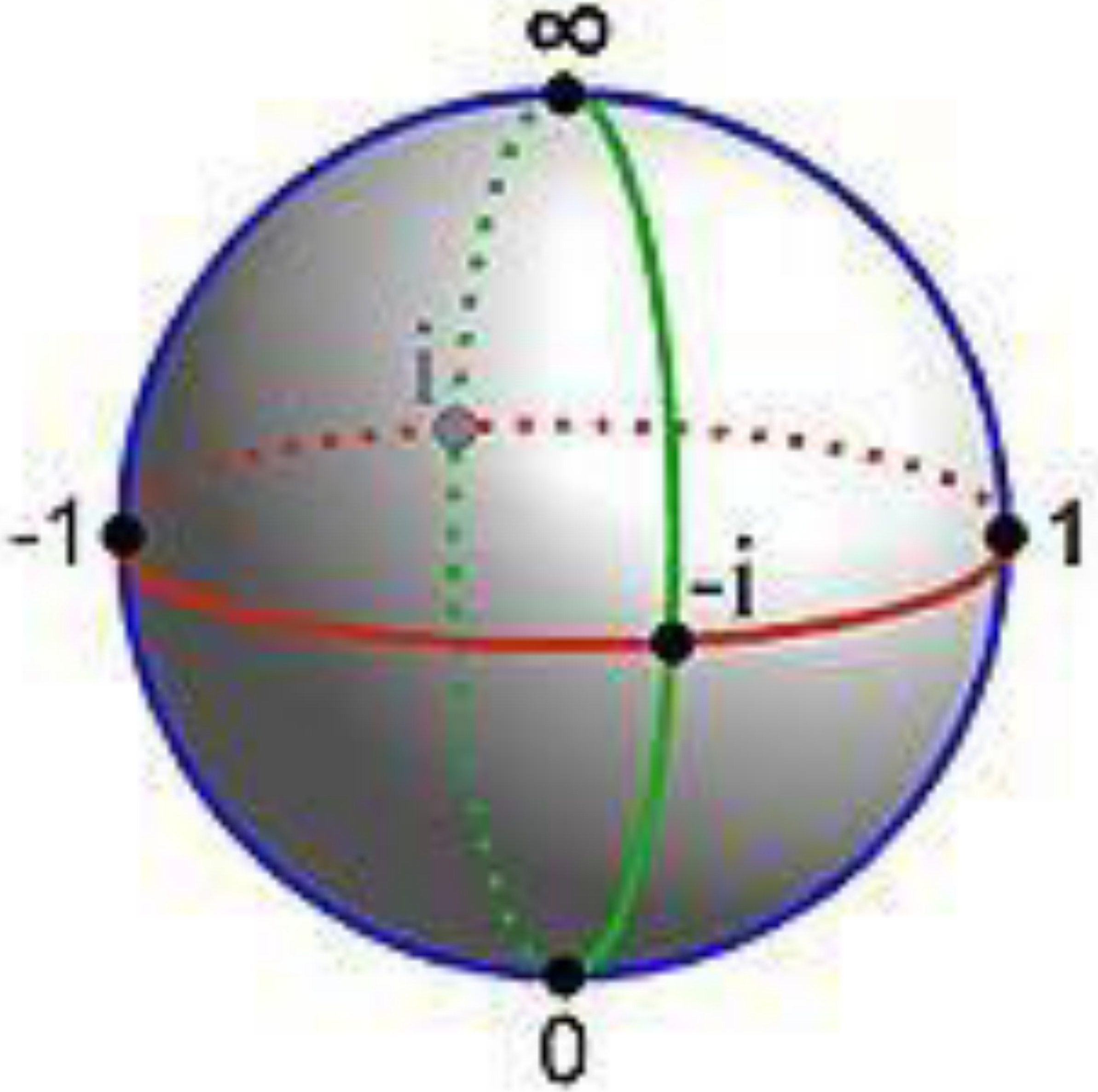
- We want manifolds to have more than just topological structure
- There should be compatibility between intersecting charts
- For intersecting charts *(U, f), (V, g),* we define a natural transition map from one set of coordinates to the other
- A manifold is C^k if it has an atlas consisting of charts with C^k transition maps
- Of particular importance are smooth maps
- Whitney embedding theorem: smooth m-manifolds can be smoothly embedded into R^{2m}

# Complex manifolds

- An n-dimensional complex manifold is a space such that, near any point *p*, each point can be defined by n complex coordinates, and the transition maps are holomorphic (complex structure)
- A manifold with n complex dimensions is also a real manifold with 2n dimensions, but with a complex structure
- Complex 1-manifolds are called Riemann surfaces (surfaces because they have 2 real dimensions)
- Holomorphic functions are "rigid" since they are defined by accumulating sets
- Whitney embedding theorem does not extend to complex manifolds

# Classifying Riemann surfaces

- 3 basic Riemann surfaces
- The complex plane
- The unit disk (distinct from plane by Liouville theorem)
- Riemann sphere (aka the extended complex plane), uses two charts
- Uniformization theorem: simply connected Riemann surfaces are conformally equivalent to one of these three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