Designing non-manipulable tournament rules

Jan Soukup, David Mikšaník
Supervisor: Ariel Schvartzman

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Motivation

- Consider the following tournament where all teams play against each other exactly once.

Tournament Results				
	A	B	C	D
A		Draw	Win	Draw
B	Draw		Loss	Win
C	Loss	Win		Win
D	Draw	Loss	Loss	

Points table	
C	6
A	5
B	4
D	1

- Suppose that the match between A and B was played as the last. Could they colluded so that one of them would won the tournament?

Motivation

- It is easy to see that they could.

Tournament Results				
	A	B	C	D
A		Win	Win	Draw
B	Loss		Loss	Win
C	Loss	Win		Win
D	Draw	Loss	Loss	

Points table	
A	7
C	6
B	3
D	1

- We would like to come up with different rules to determine winners of tournaments that is "reasonable" and teams don't have any incentive to collude.

Our model

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A *tournament* T on n teams is a complete directed graph with n vertices (we do not allow draws).

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Theorem

There exists a rule that is (CC) and (2-SNM-1/3) and no better rule exists.

Our goals

- Generally, we consider the case when k teams collude:
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- Our goal is to improve the lower bound on α (if possible), for some $k \geq 3$, and find a rule that achieve this lower bound.
- We will try to come up with other reasonable ways how to relax (CC) and (k-SNM) properties.