Designing non-manipulable tournament rules

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Supervisor: Ariel Schvartzman

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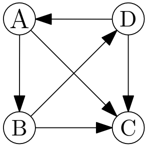
Introduction: an example

- Given $\binom{n}{2}$ outcomes between n teams ($=$ tournament), determine the winner of the tournament ($=$ tournament rule)
- Example: random single binary elimination bracket (RS2EB)

$T =$ 

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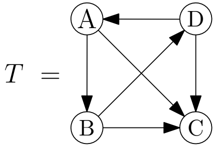

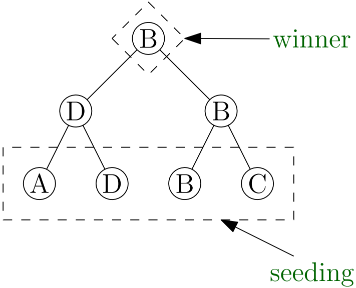

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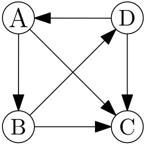
$T =$ 

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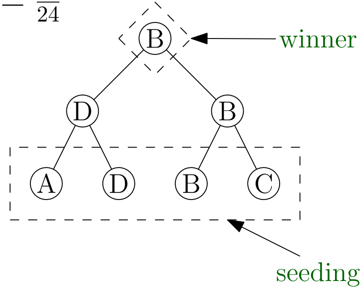

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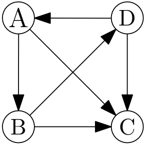

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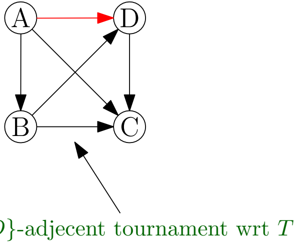

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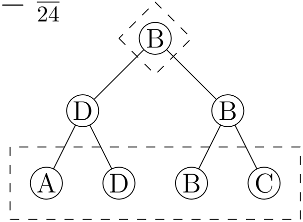

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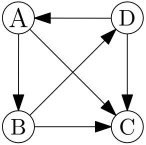
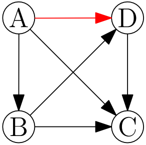
$T =$ 

$T' =$ 

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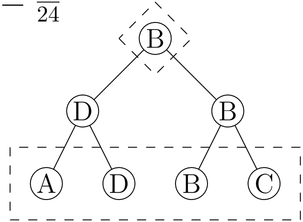

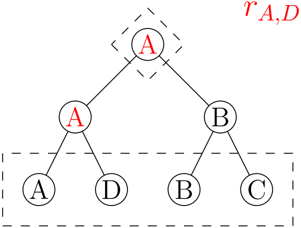

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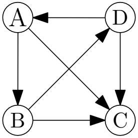
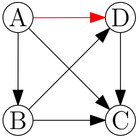
$T =$ 

$T' =$ 

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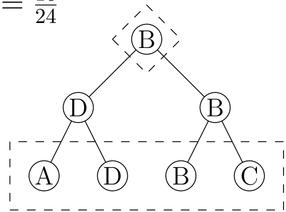

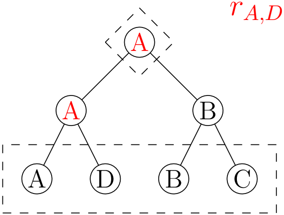

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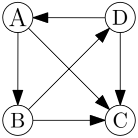
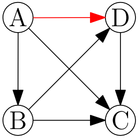
$T =$ 

$T' =$ 

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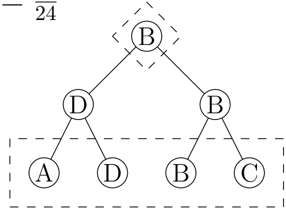

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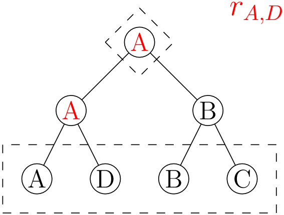

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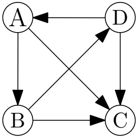
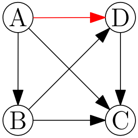
$T =$ 

$T' =$ 

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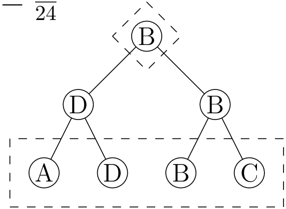

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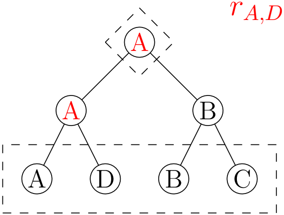

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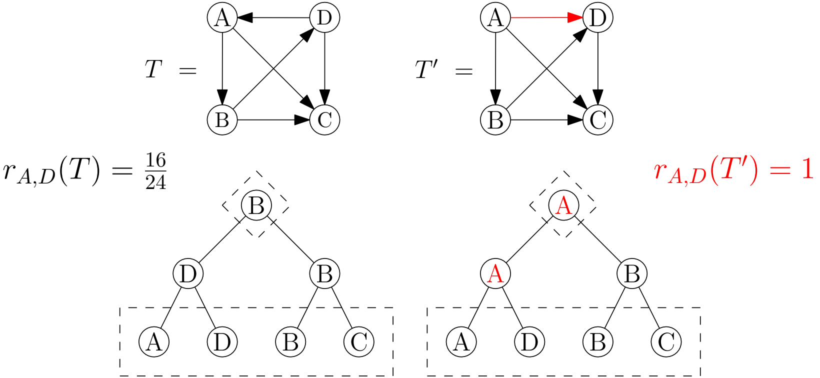

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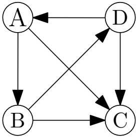

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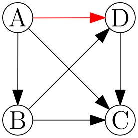

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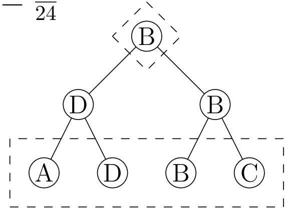

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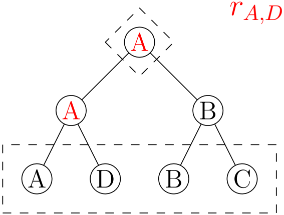

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- Fact: $\alpha = \frac{1}{3}$ *(proved by Altman and Kleinberg [1])*
- We say that the rule RS2EB is 2-SNM-$\frac{1}{3}$

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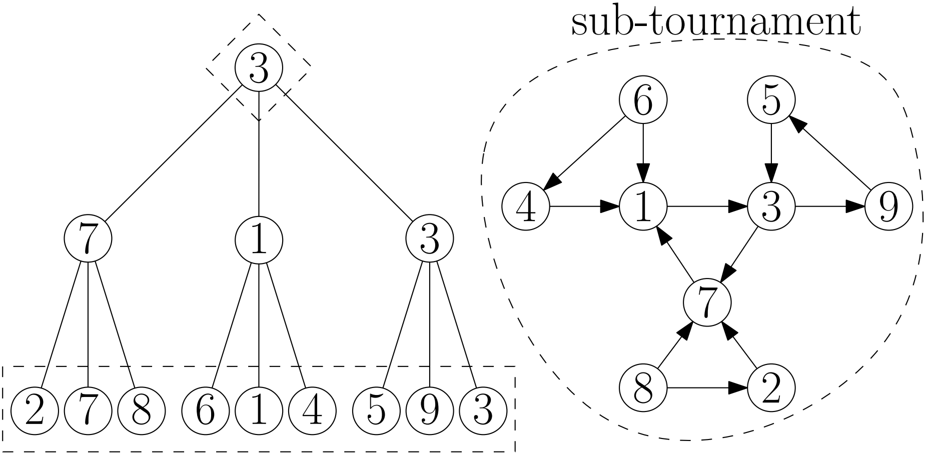
Theorem (Dinev and Weinberg, 2022; not published yet)

There exists a rule that is monotone, (CC), (2-SNM-$\frac{1}{3}$), and (3-SNM-$\frac{31}{60}$).

Random single ternary elimination bracket (RS3EB)



Theorem

The rule RS3EB is monotone, (CC), and (3-SNM-α) for $\frac{31}{60} < \frac{227}{420} \leq \alpha \leq \frac{23}{27}$.

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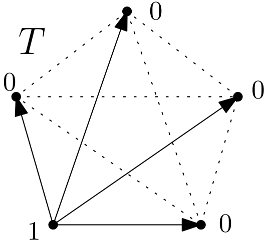
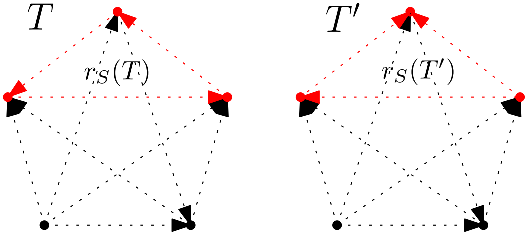


$r_a(T) = 1$

(k-SNM-α) condition

- Impacts only specific pairs of tournaments



$r_S(T') \leq r_S(T) + \alpha$

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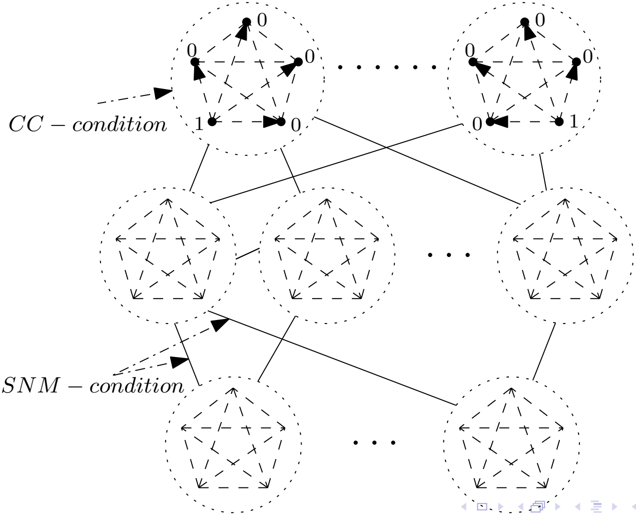

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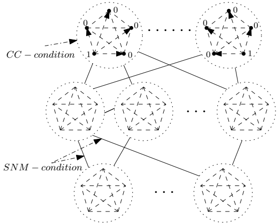
- We have restrictions imposed on a subset of vertices and a subset of edges
- If a tournament is not adjacent to a tournament with restriction (imposed by (CC) property) we distribute the probability uniformly
- The remaining tournaments (the middle layer) need a bit more work



Theorem

There is a tournament rule that is monotone, (CC), (2-SNM-$\frac{1}{3}$), and (3-SNM-$\frac{1}{2}$).

- Previously, there was no known rule that was both (CC) and (3-SNM-α) for $\alpha < \frac{31}{60}$.

Rule extensions

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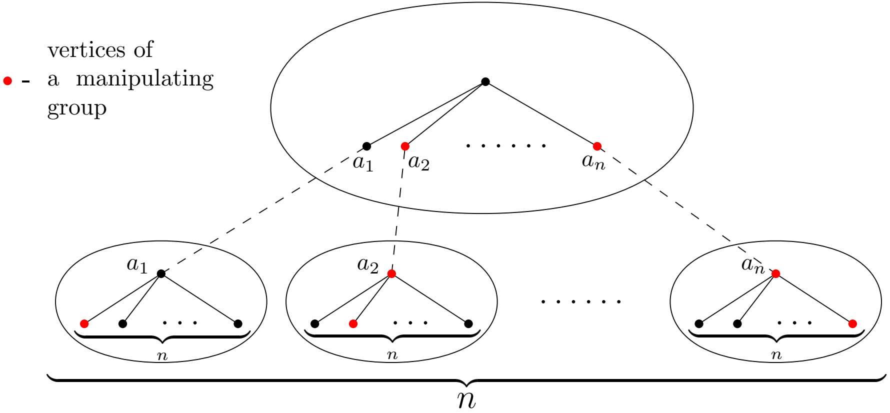
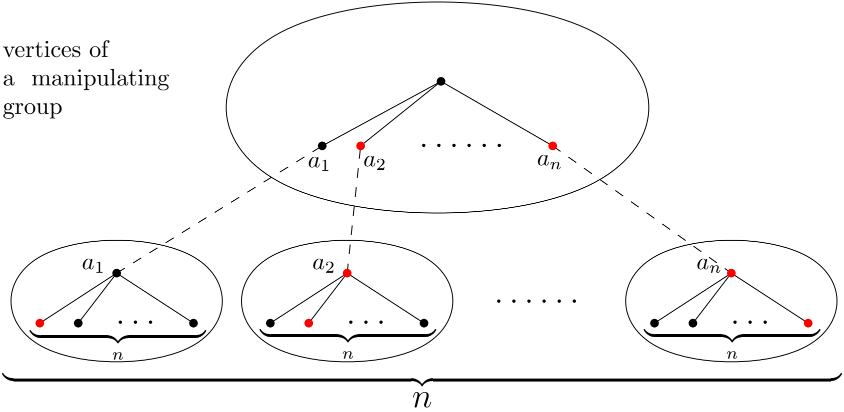


● - vertices of a manipulating group



- This extended rule is still (CC)
- If the size of a manipulating group is a lot smaller than n then this rule is (k-SNM-$(\alpha + \varepsilon)$) if the original rule is (k-SNM-(α))

References

[1] Altman, A., Kleinberg, R.: Nonmanipulable randomized tournament selections. Proceedings of the National Conference on Artificial Intelligence **2**, 686–690 (01 2010)

[2] Schvartzman, A., Weinberg, S.M., Zlatin, E., Zuo, A.: Approximately strategyproof tournament rules: on large manipulating sets and cover-consistence. In: 11th Innovations in Theoretical Computer Science Conference, LIPIcs. Leibniz Int. Proc. Inform., vol. 151, pp. Art. No. 3, 25, Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern (2020)

Thank you for your attention!