Graph theory and Brain connectivity

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A graph is a non empty set of vertices (or nodes) \( V \) and a set of edges \( E \) that connect pairs of nodes.
Directed Graph

- A directed graph is a graph where the vertices are joined by directed edges.
Undirected Graph

- An undirected graph is a graph where the nodes are connected with undirected edges.
A graph can be weighted or unweighted. A weighted graph is a graph where the edges have weights assigned to them.
Degree

- Undirected graph: the number of edges connected to a node
- Directed graph: The *in-degree of a vertex* $v$, denoted $\text{deg}^-(v)$, is the number of edges directed into $v$.
- The *out-degree of* $v$, denoted $\text{deg}^+(v)$, is the number of edges directed out of $v$. 
Clustering coefficient

• $C_i = \frac{\# \text{ of connections between neighbors of } i}{\# \text{ of potential connections}}$

$C_i = \frac{1}{6}$
Path length

• Unweighted: the number of edges between two nodes
  – Weighted: weights of edges must be taken into consideration

• Often we want to find the shortest path
Betweenness

• Consider all pairs of distinct nodes in a graph and determine the shortest path length between them. The betweenness of a node is the frequency with which it occurs in all shortest paths.
Edge density

- $E = \frac{\text{# of existing edges}}{\text{# of potential connections}}$

$E = \frac{6}{15} = 0.4$
Adjacency matrix
• Adjacency matrix $\rightarrow$ Laplacian $\rightarrow$ Normalized Laplacian $\rightarrow$ $\rightarrow$ eigenvalues, eigenvectors for spectral analysis
Application of Graph

• We can say that almost “EVERYTHING IS A GRAPH”
• Graph theory is used today in the modelling of:
  • Social networks
  • Communications networks
  • Information networks
  • Software design
  • Transportation networks
  • Biological networks
  • Etc.
## Viewing brain connectivity as a graph

<table>
<thead>
<tr>
<th></th>
<th>Functional connectivity</th>
<th>Structural connectivity</th>
<th>Effective connectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nodes</strong></td>
<td>Functional regions of interest (ROIs)</td>
<td>Neurons</td>
<td>ROIs</td>
</tr>
<tr>
<td><strong>Edges</strong></td>
<td>Fiber tracts</td>
<td>Axons</td>
<td>Causal relationship</td>
</tr>
</tbody>
</table>

**How are we going to define connectivity?**
**How are functional, structural, and effective connectivity related?**
Networks

- Random
- Ordered
- Scale-free
- Small-world

- How do clustering coefficients and path lengths vary between functional and structural analyses?
Our challenge:

Box 1: Structural and functional brain networks

- Histological or imaging data
- Anatomical parcellation
- Recording sites
  - Time series data
  - Structural brain network
  - Functional brain network (Sensorimotor, Premotor)
- Graph theoretical analysis

Regions: Occipital, Parietal, Inferior temporal, Orbitofrontal, Prefrontal, Temporal pole
Ideas:

- Structural connections of the human brain
  - Sex differences
  - Epilepsy, seizures
  - Brain damage/impairment
  - Memory and higher cognition
Sources:

• Stam, Cornelis, and Jaap Reijneveld. “Graph theoretical analysis of complex networks in the brain.” *Nonlinear Biomedical Physics* 1:3. n. pag. Print.