Entire List Coloring

David Lapayowker
Advisor: Dr. Cranston

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Graph Coloring, A Review

A **Graph** consists of a series of vertices connected by edges.

To **color** a graph means to assign a color to every vertex such that no two vertices that share an edge also share a color.

Many applications of graph coloring call for finding the smallest number of colors necessary to color the graph. This is the **chromatic number**, and is denoted $\chi$. 
An *Entire Coloring* is much like a vertex coloring, except you also color all the edges and faces of the graph, so that no two adjacent elements share a color. This is denoted $\chi_{vef}$.

**Figure:** Example of entire coloring. Image courtesy of www.ams.org
Entire Coloring

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![Example of entire coloring](image)

**Figure:** Example of entire coloring. Image courtesy of [www.ams.org](http://www.ams.org)

For all graphs $G$, $\chi_{vef}(G) > \chi(G)$. 
A *List Coloring* is a type of coloring in which every vertex must be colored from a specified list, unique to each vertex. This is denoted $\chi^L$. 
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A graph is *k list colorable* if, given that every vertex has a list of colors of size $k$, it is always possible to choose a color in each vertex’s color list to properly color the graph *no matter what those lists contain*. 
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David Lapayowker  Advisor: Dr. Cranston  ()
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However, a similar statement can only be made for $\chi^L_{vef}(G)$ if $\Delta \geq 12$. 
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However, a similar statement can only be made for $\chi_{vef}^L(G)$ if $\Delta \geq 12$.

We wanted to see if we could translate arguments about entire colorings into entire list colorings, and improve on the second lower bound.
Theorem

All planar graphs are 6-list colorable.
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Lemma

All planar graphs have a vertex of degree 5 or less.

Proof follows from Euler’s formula for planar graphs.
Example: 6-List Colorability

Find this 5^- vertex, and remove it from the graph.
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Find this $5^-$ vertex, and remove it from the graph.

By induction, color all remaining vertices from their lists.
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Find this 5− vertex, and remove it from the graph.

By induction, color all remaining vertices from their lists.

Add the removed vertex back and color it from its list.
Questions?