Bayesian Statistical Modeling: Gaussian Process Regression Applied to Sea-Level Data

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Brief Description

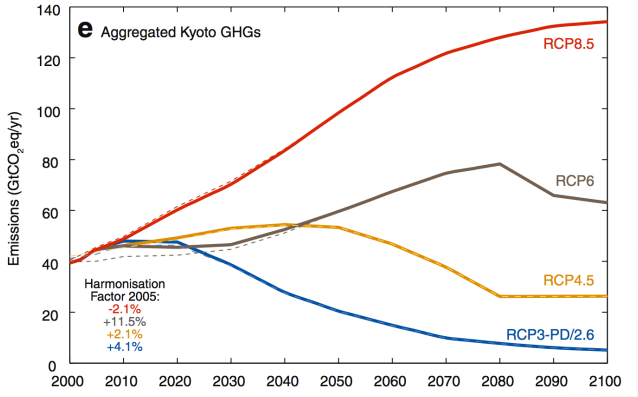
- I used **Gaussian process regression** to model relative sea level
- Robert DeConto and David Pollard modeled global mean sea level (GMSL) using measures which quantify different oceanic and atmospheric processes [1]
  - Emulate their model to decrease run time and get a full posterior distribution
- Effectively visualize emulator results

Data

Parameter	Description
OCFAC	Sub-ice oceanic melt
CREVLIQ	Hydrofracturing due to surface liquid
VCLIF	Ice-cliff structural failure

Response	Description
PGMSL	Pliocene global mean sea level
LIGGMSL	Last Interglacial global mean sea level

RCP	Description
2.6	Slight increase in emissions before an early decline
4.5	Emissions rise at first, but then begin to decline part of the way through the 21st century
8.5	"Worst-case scenario"; emissions continue to increase

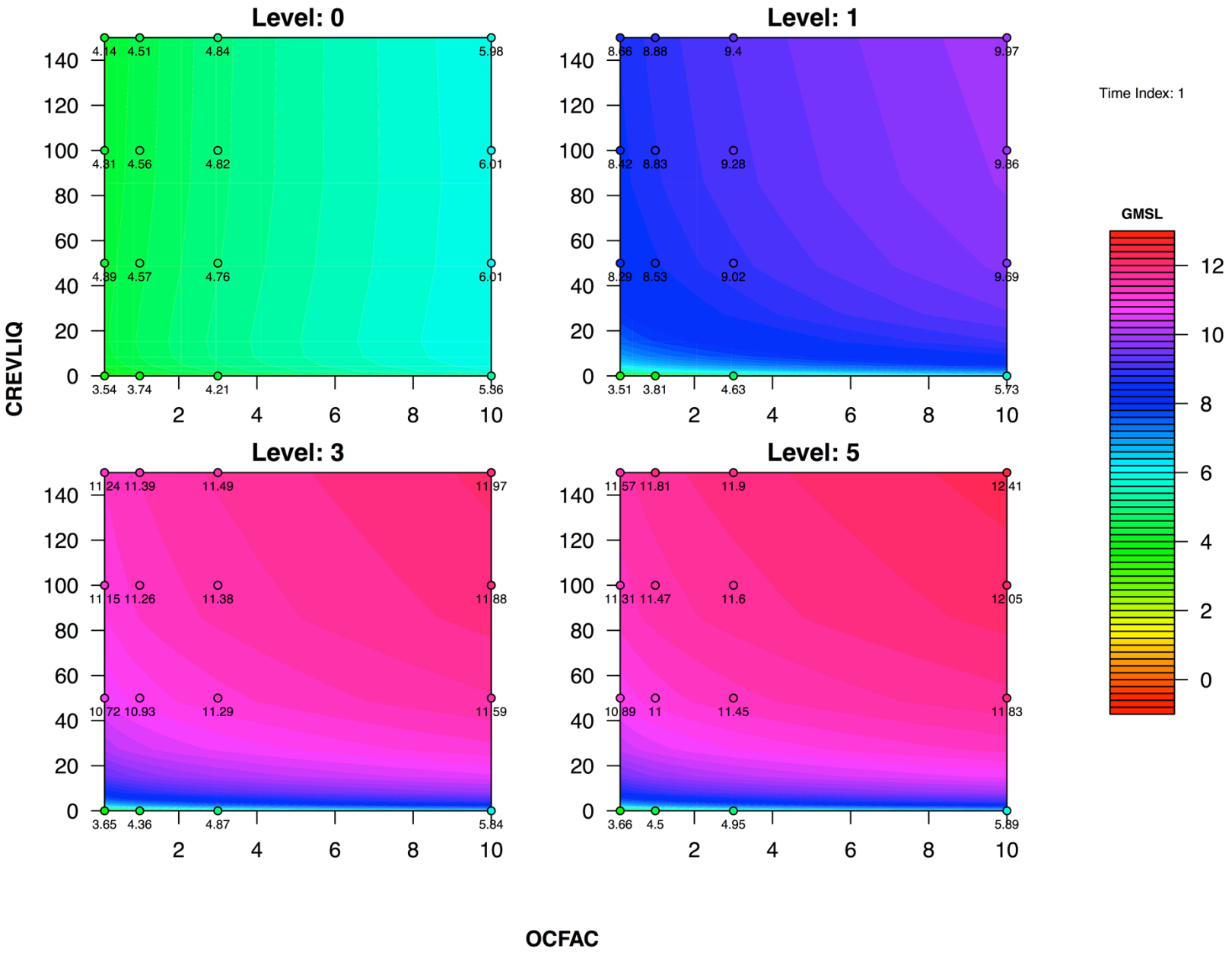


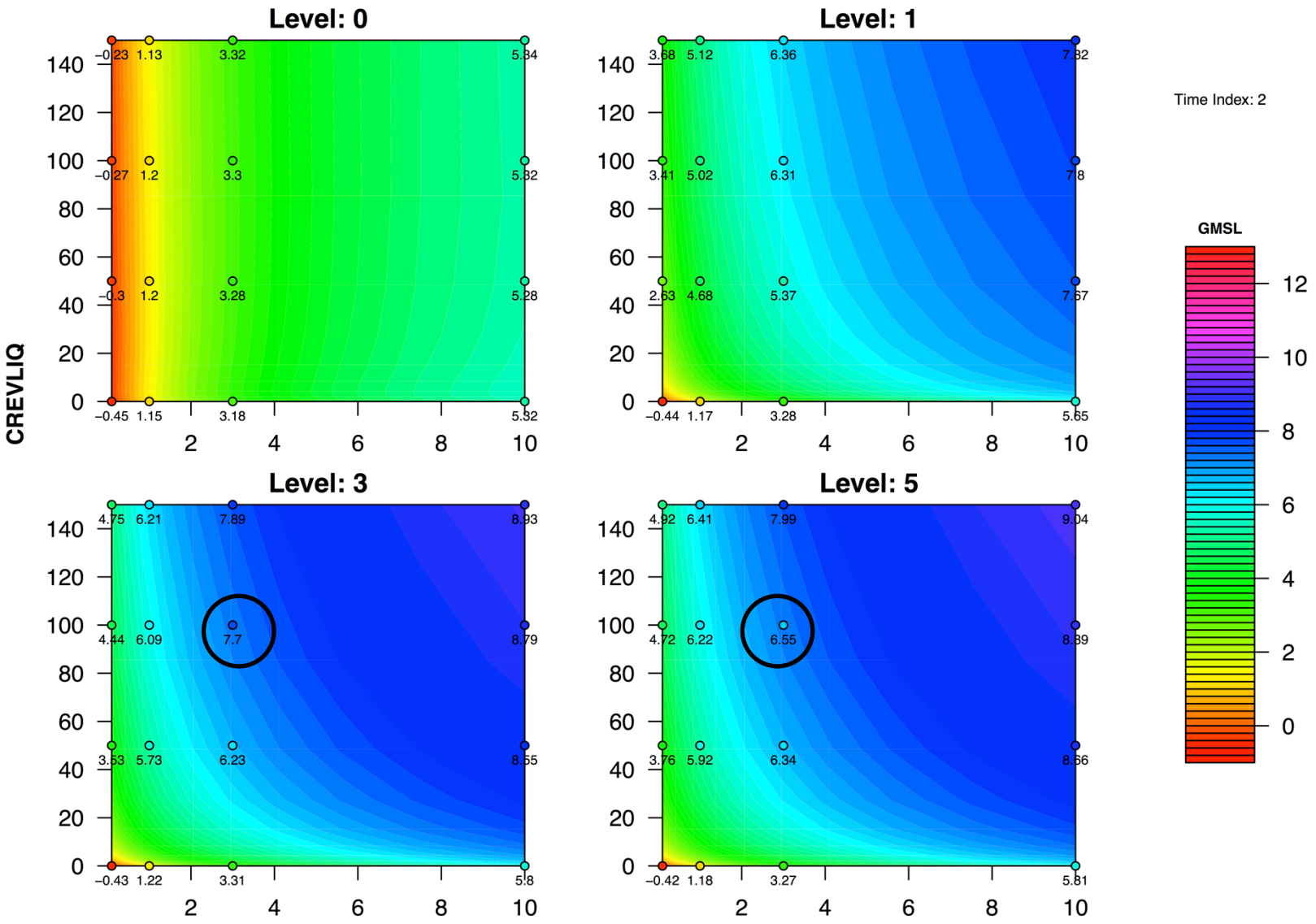
Overview

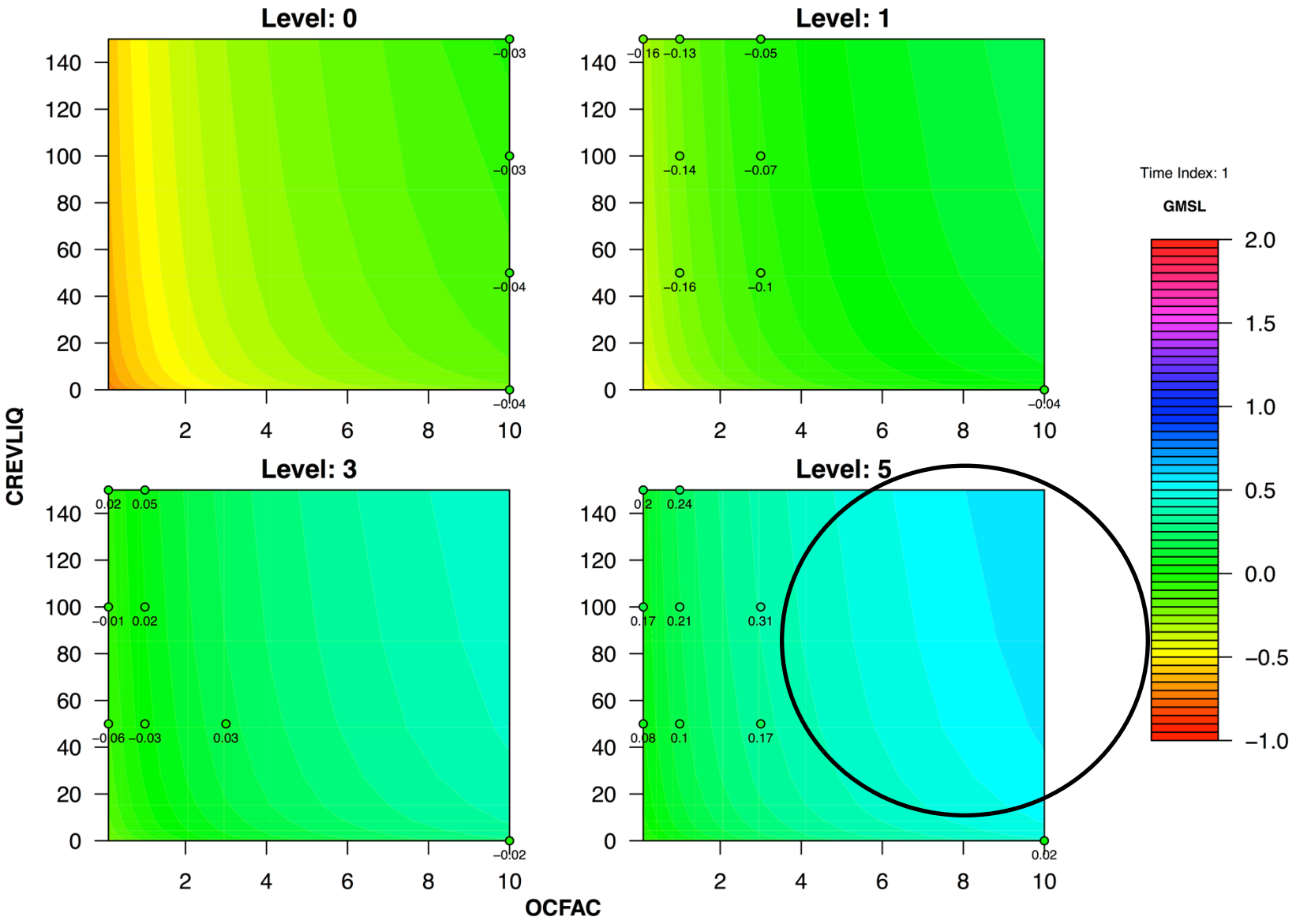
- **Visualize** data
- Build **emulators** for:
  - Pliocene and Last Interglacial GMSL
  - GMSL for each RCP (2.6, 4.5, and 8.5) separately

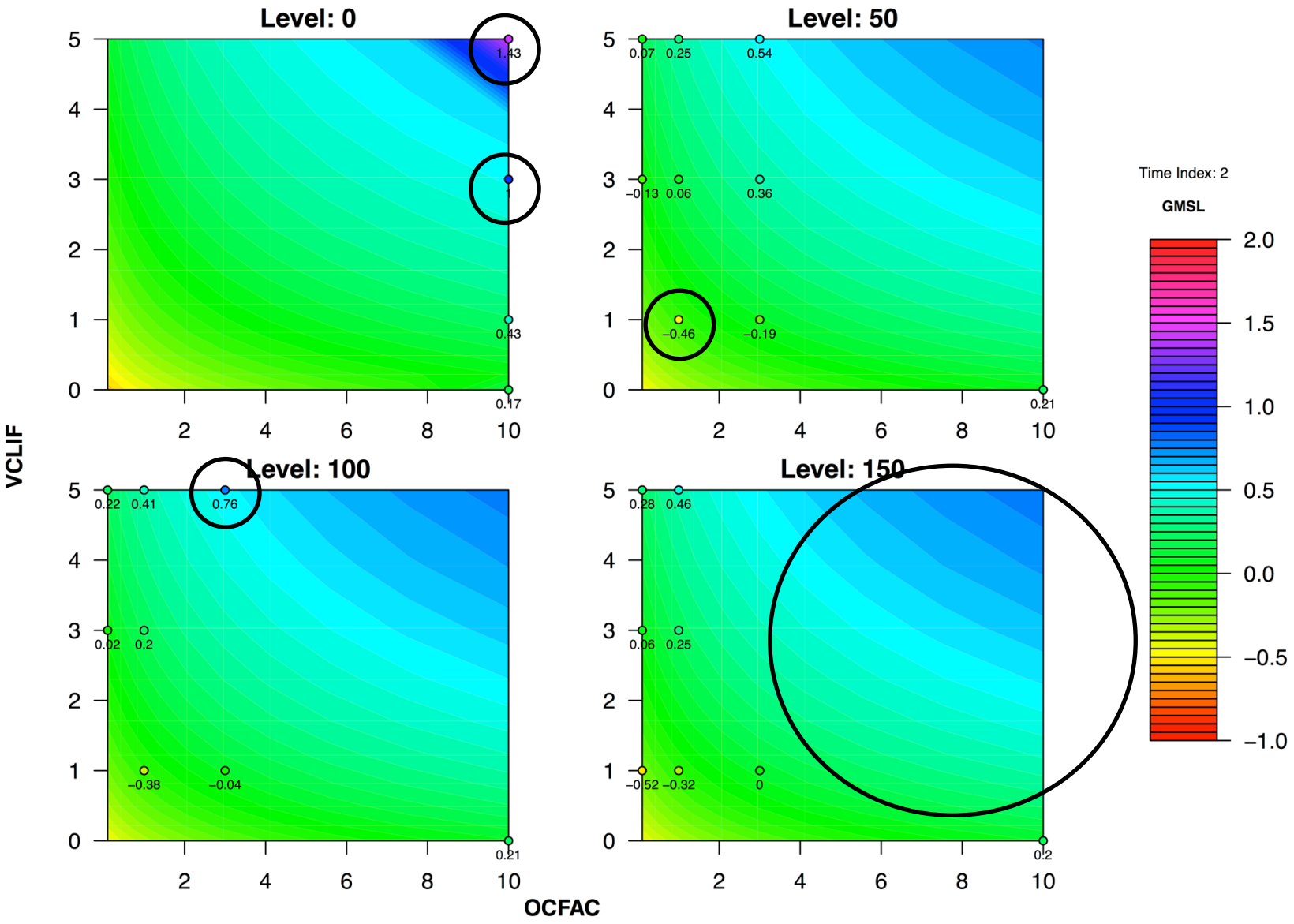
  using data from simulations of a model by DeConto and Pollard
- **Visualize** emulator GMSL predictions and error through various plots and time series

GMSL Contour Plots

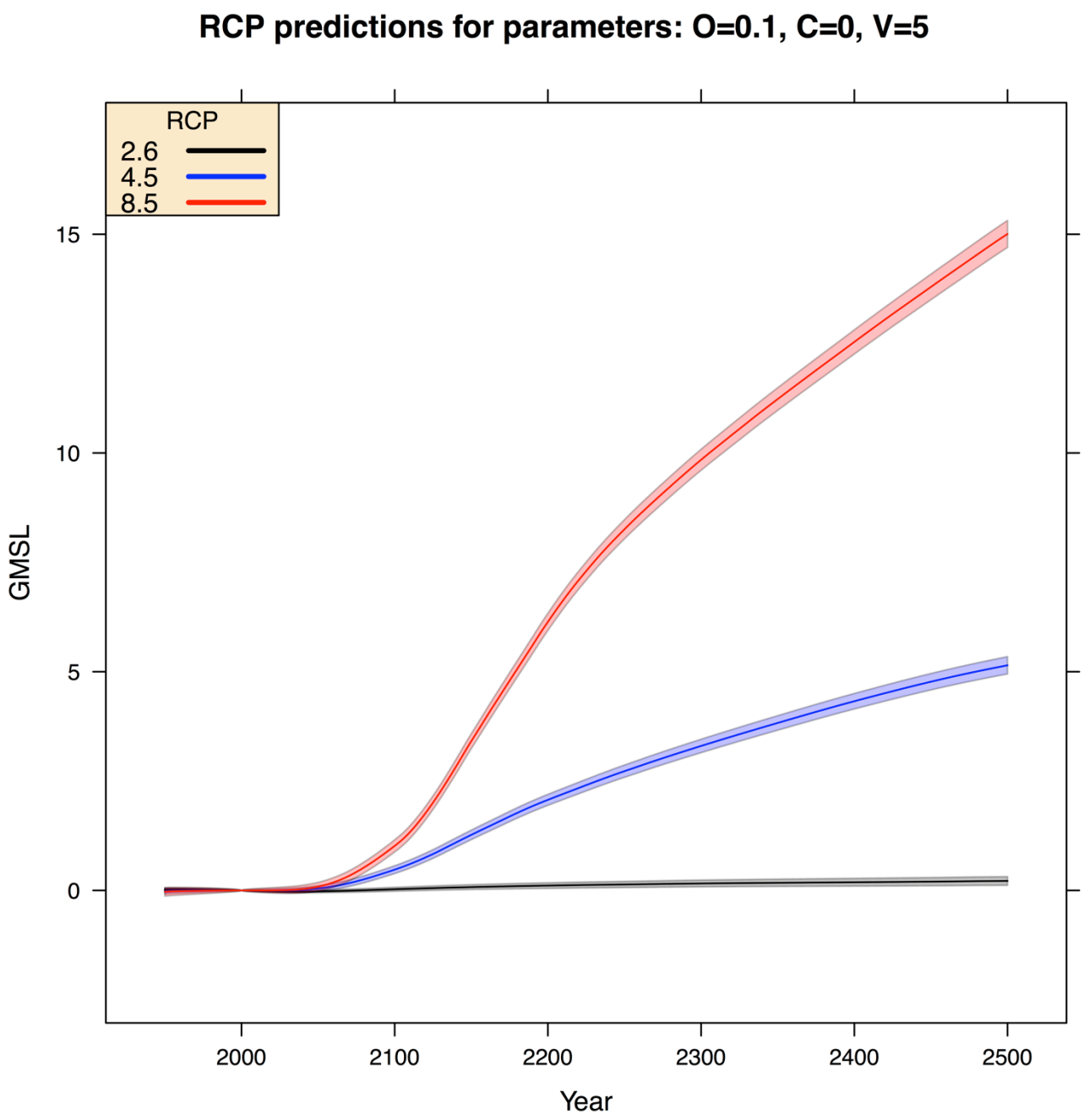

Level: 0
Level: 1
Level: 3
Level: 5
Time Index: 1
GMSL
CREVLIQ
OCFAC
4.14 4.51 4.84 5.98
4.31 4.56 4.82 6.01
4.39 4.57 4.76 6.01
3.54 3.74 4.21 5.36
8.66 8.88 9.4 9.97
8.42 8.83 9.28 9.86
8.29 8.53 9.02 9.69
3.51 3.81 4.63 5.73
11.24 11.39 11.49 11.97
11.15 11.26 11.38 11.88
10.72 10.93 11.29 11.59
3.65 4.36 4.87 5.84
11.57 11.81 11.9 12.41
11.31 11.47 11.6 12.05
10.89 11 11.45 11.83
3.66 4.5 4.95 5.89

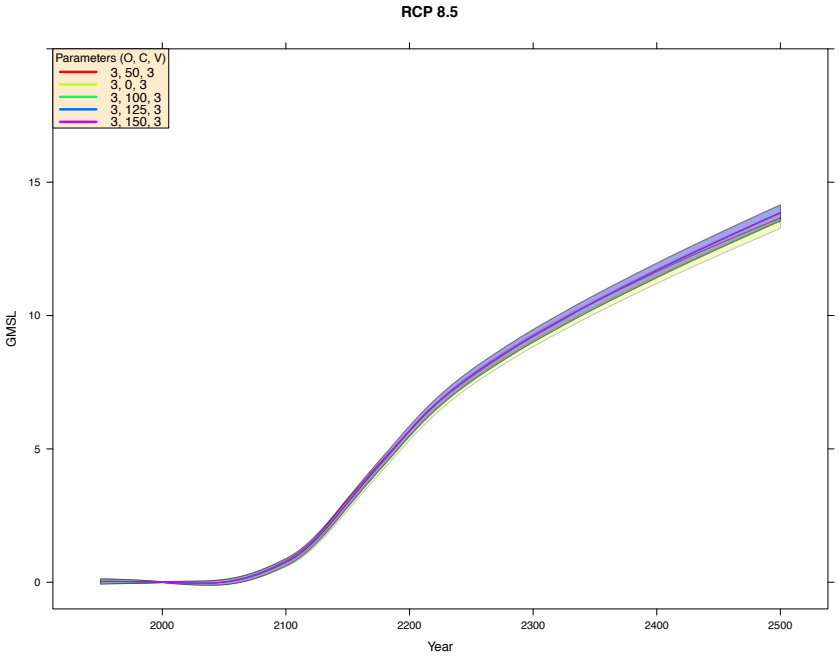

Level: 0
Level: 1
Level: 3
Level: 5
Time Index: 2
GMSL
CREVLIQ
OCFAC

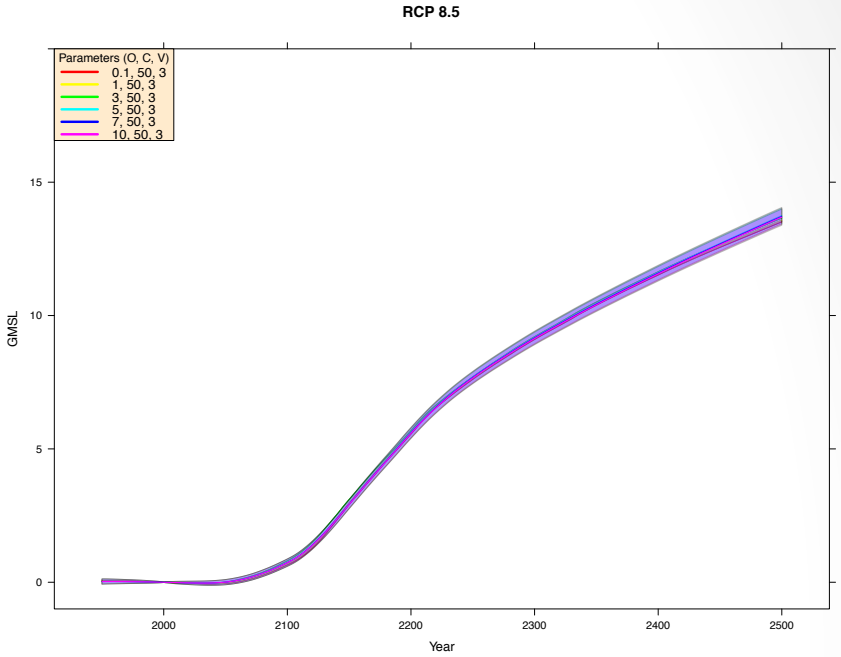

Level: 0
Level: 1
Level: 3
Level: 5
CREVLIQ
OCFAC
Time Index: 1
GMSL
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0

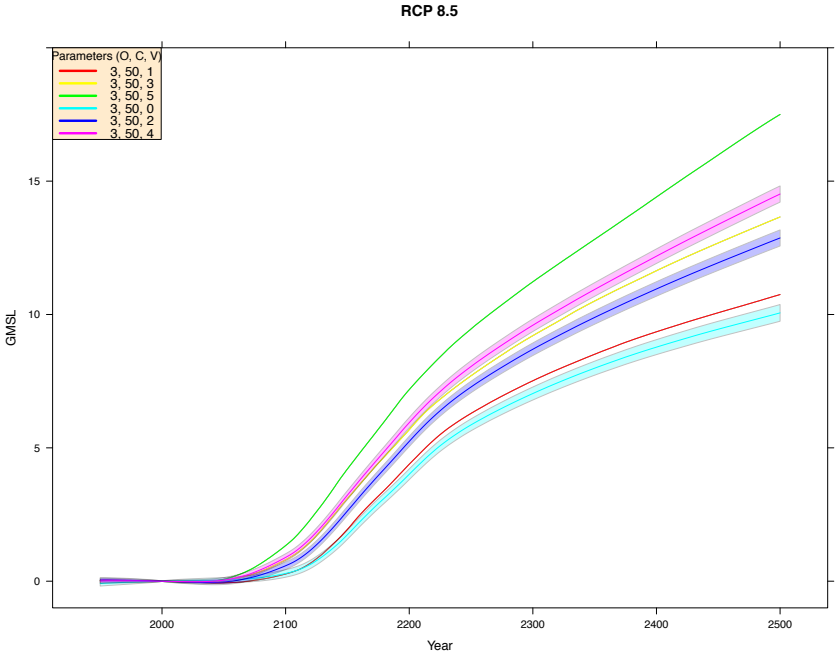

Level: 0
1.43
0.43
0.17
Level: 50
0.07
0.25
0.54
−0.13
0.06
0.36
−0.46
−0.19
0.21
Level: 100
0.22
0.41
0.76
0.02
0.2
−0.38
−0.04
0.21
Level: 150
0.28
0.46
0.06
0.25
−0.52
−0.32
0
0.2
Time Index: 2
GMSL
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
VCLIF
OCFAC
0
1
2
3
4
5
2
4
6
8
10

Time Series Plots

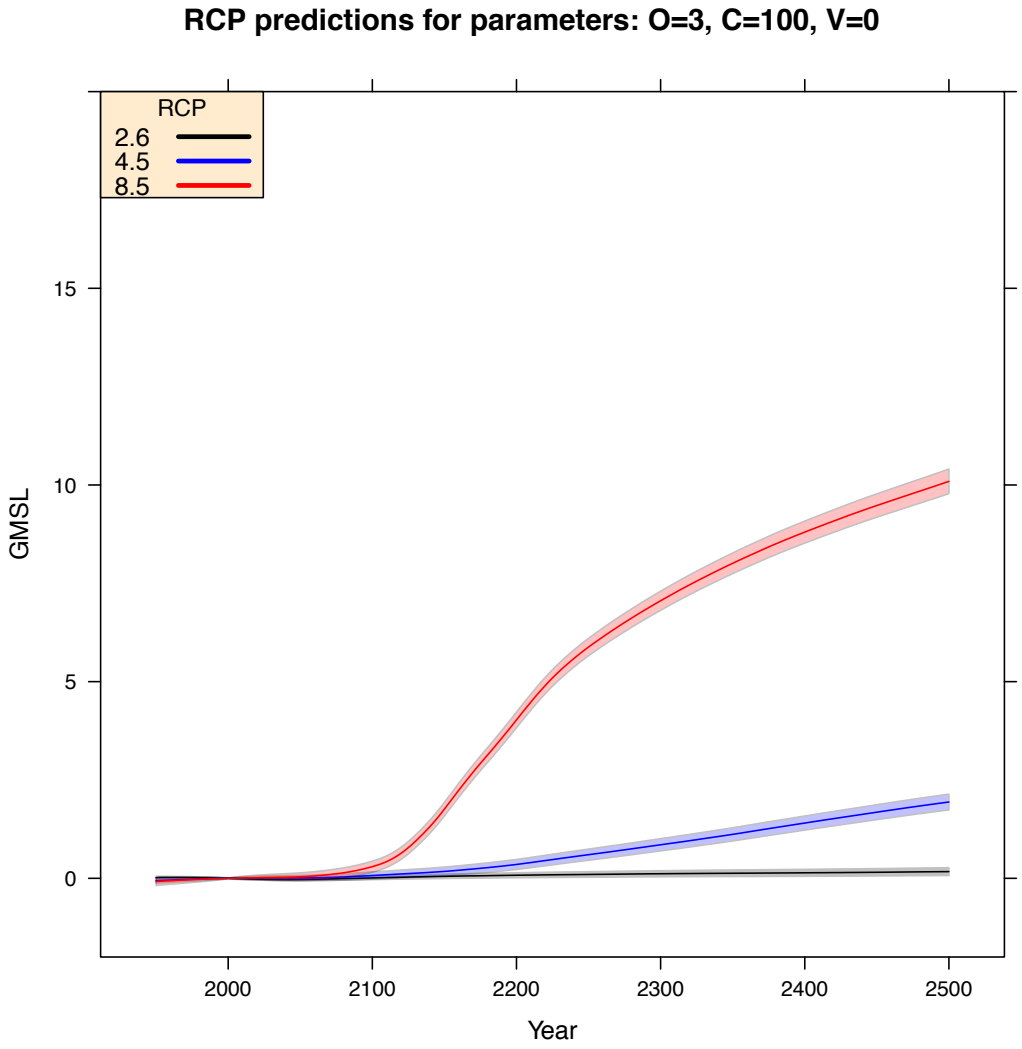

RCP predictions for parameters: O=0.1, C=0, V=5
RCP
2.6
4.5
8.5
GMSL
15
10
5
0
2000
2100
2200
2300
2400
2500
Year

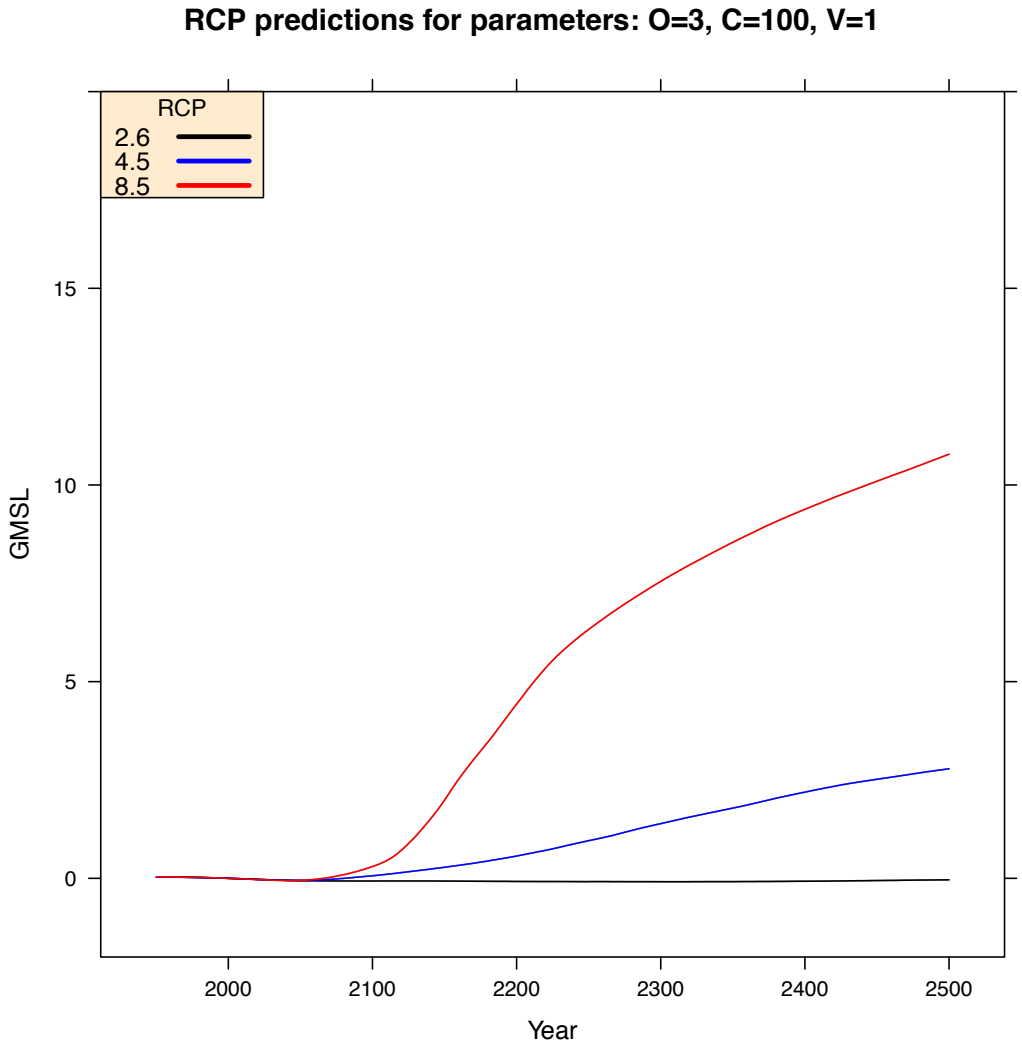

RCP 8.5
Parameters (O, C, V)
3, 50, 3
3, 0, 3
3, 100, 3
3, 125, 3
3, 150, 3
GMSL
Year
0
5
10
15
2000
2100
2200
2300
2400
2500

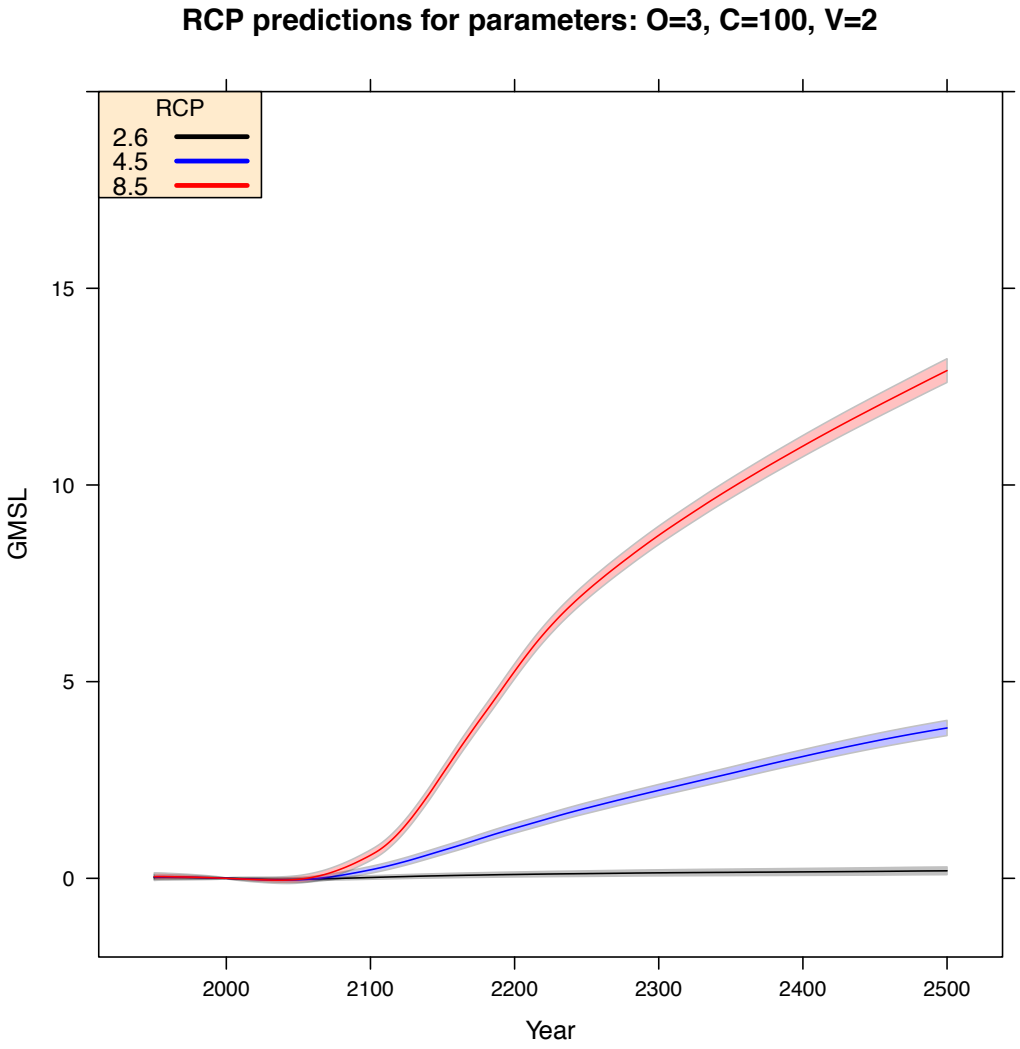

RCP 8.5
Parameters (O, C, V)
0.1, 50, 3
1, 50, 3
3, 50, 3
5, 50, 3
7, 50, 3
10, 50, 3
GMSL
Year
0
5
10
15
2000
2100
2200
2300
2400
2500

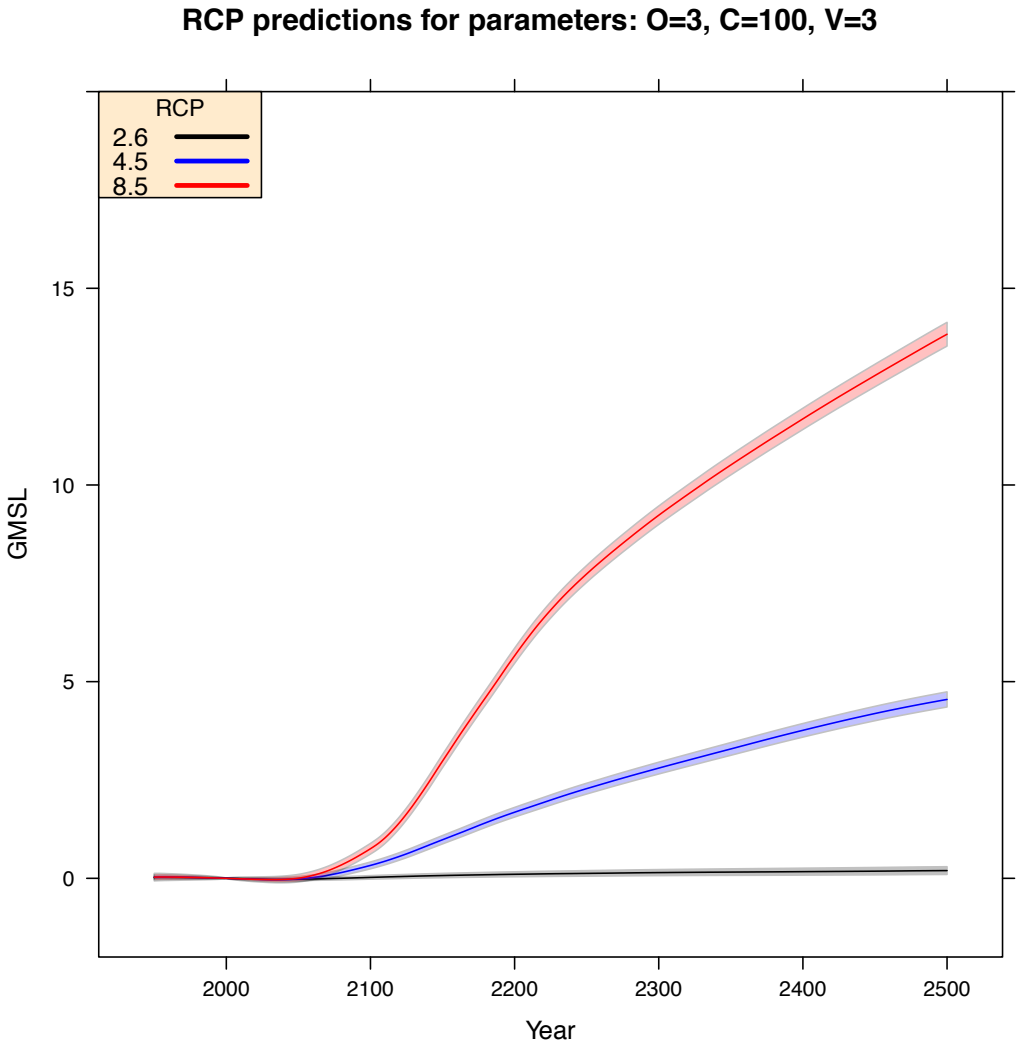

RCP 8.5
Parameters (O, C, V)
3, 50, 1
3, 50, 3
3, 50, 5
3, 50, 0
3, 50, 2
3, 50, 4
GMSL
Year
0
5
10
15
2000
2100
2200
2300
2400
2500

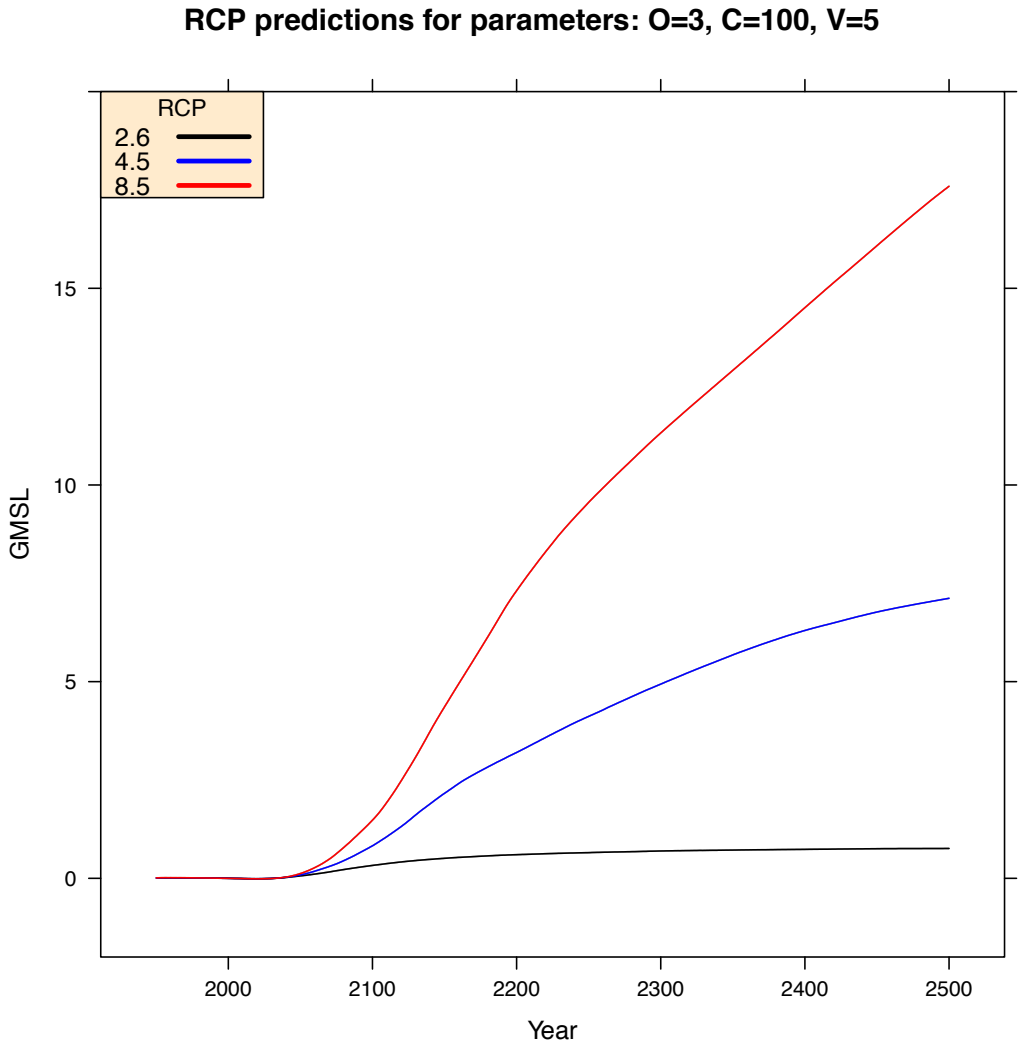
Time Series Progression


RCP predictions for parameters: O=3, C=100, V=0
RCP
2.6
4.5
8.5
GMSL
15
10
5
0
2000
2100
2200
2300
2400
2500
Year


RCP predictions for parameters: O=3, C=100, V=1
RCP
2.6
4.5
8.5
GMSL
0
5
10
15
2000
2100
2200
2300
2400
2500
Year

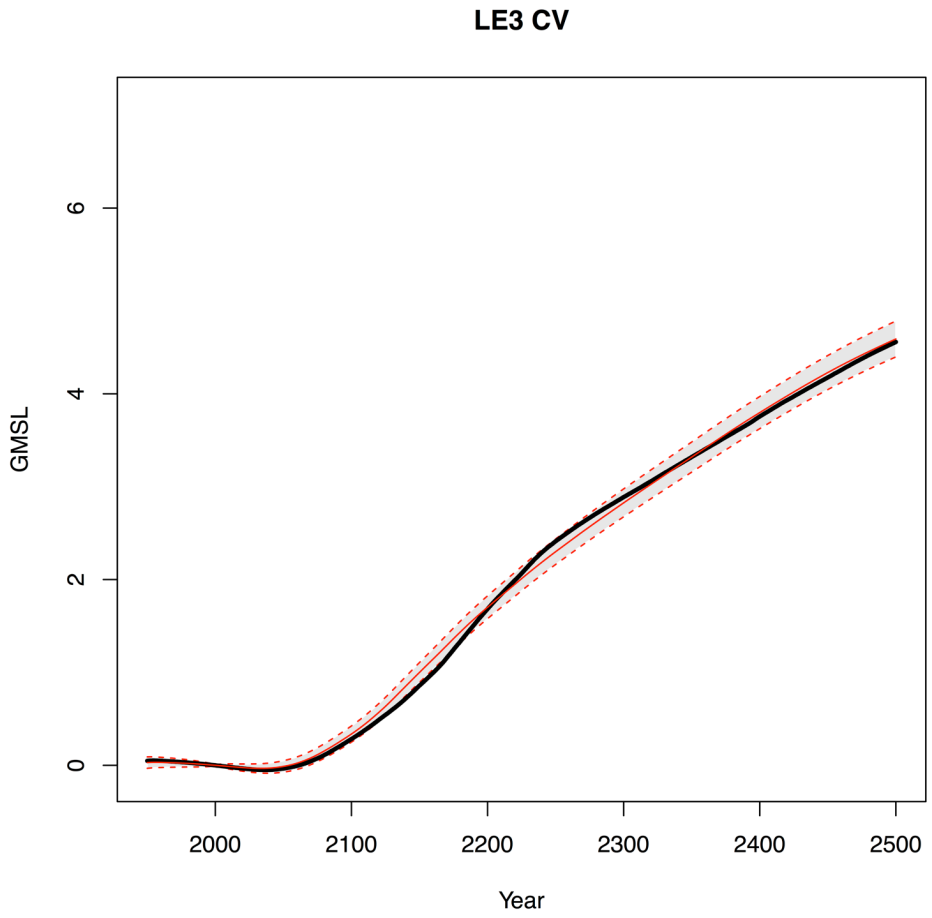

RCP predictions for parameters: O=3, C=100, V=2
RCP
2.6
4.5
8.5
GMSL
0
5
10
15
2000
2100
2200
2300
2400
2500
Year


RCP predictions for parameters: O=3, C=100, V=3
RCP
2.6
4.5
8.5
GMSL
0
5
10
15
2000
2100
2200
2300
2400
2500
Year

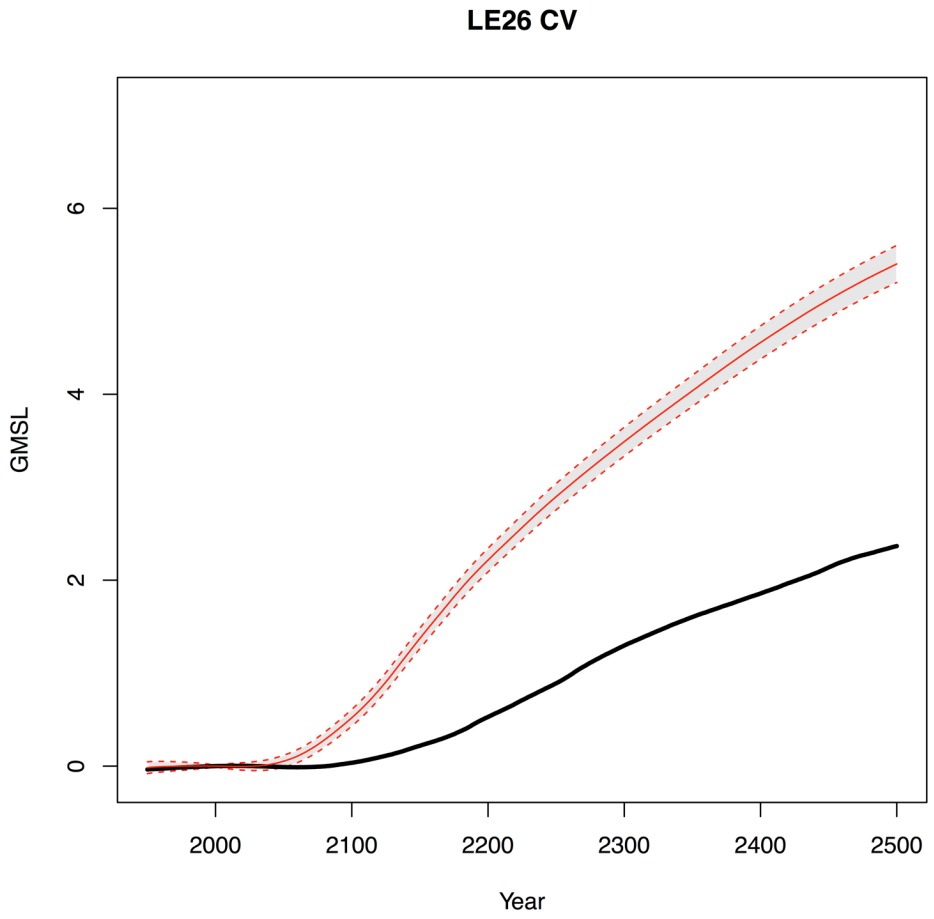

RCP predictions for parameters: O=3, C=100, V=5
RCP
2.6
4.5
8.5
GMSL
0
5
10
15
2000
2100
2200
2300
2400
2500
Year

Cross Validation

A good result:



A not-so-good result:



Next Steps

- Look at more **covariance functions** to improve emulator results
- Further **generalize code** to run with other sea-level data
- Make **future predictions** for times outside of the data range

Sources

[1] DeConto, R.M. and D. Pollard. Contribution of Antarctica to past and future sea-level rise. *Nature* 531, 591-597 (2016).

[2] Meinshausen, M. *et al*. The RCP greenhouse gas concentrations and their extensions from 1765 to 2300. *Climatic Change* 109, 213-241 (2011).

[3] Olson R. and W. Chang. Mathematical framework for a separable Gaussian Process Emulator (2013).

[4] Rasmussen C.E. and C.K.I. Williams. Gaussian Processes for Machine Learning. MIT Press, 2006.

[5] Roman Olson, Won Chang, Klaus Keller and Murali Haran (2014). stilt: Separable Gaussian Process Interpolation (Emulation). R package version 1.0.1.

Thank you!

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