DIMACS: Schubert Calculus

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Project

* Proving an identity about curve neighborhoods in the Grassmannian
Grassmannian

• Gr(m,n) is the set of all m-dimensional subspaces in a vector space of dimension n
• GL(n) is the set of all invertible n x n matrices
• Borel subgroup is a subgroup of GL(n) which is the set of all upper triangular matrices
• Maximal Tori is a subgroup of the Borel subgroup which is the set of all diagonal matrices
Schubert Calculus

• Schubert Calculus is the manipulation of Schubert cells
• Schubert cells are orbits of the action of the Borel subgroup on \( \text{Gr}(m,n) \)
• Schubert variety is the closure of the Schubert cells
Curve Neighborhoods

• Let $X = \text{Gr}(m,n)$
• $\Omega \leq X$, closed subset
• $\Gamma_d(\Omega) –$ closure of the union of all curves of degree $d$ in $X$ that intersect $\Omega$
Curve Neighborhoods (cont.)

• Take the curve neighborhood of a Schubert variety to obtain a new Schubert variety of higher dimension

• How are these two varieties related?