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Rainbow cycles

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Rainbow cycle

- Graph $G(V, E)$
  - n vertices, m edges
- Colors $\{1, \ldots, n\}$
- Rainbow cycle R
  - each e has color $c(e)$ in $\{1, \ldots, n\}$
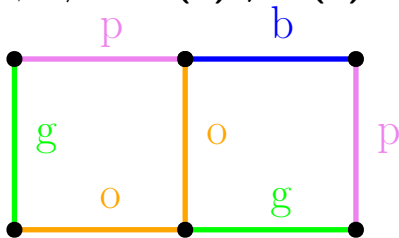 - $\forall e, f \in R, e \neq f : c(e) \neq c(f)$

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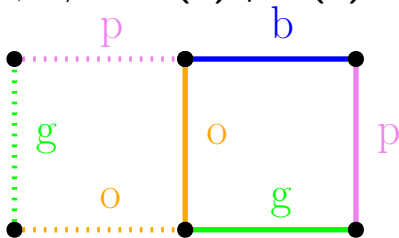 - $\forall e, f \in R, e \neq f : c(e) \neq c(f)$

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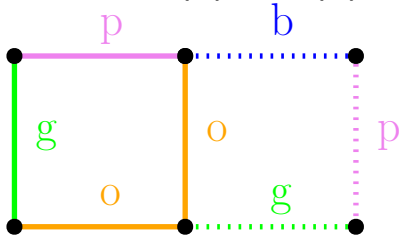 - $\forall e, f \in R, e \neq f : c(e) \neq c(f)$

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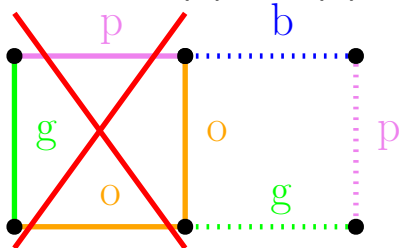 - $\forall e, f \in R, e \neq f : c(e) \neq c(f)$

Main problem

- Graph on n vertices
- Sets of colors $\{E_1, E_2, \ldots, E_n\}$
- For each color $i : |E_i| \geq k$

Question

Exists in G a rainbow cycle
of length at most $\lceil \frac{n}{k} \rceil$?

Example

- n ... num. of vertices
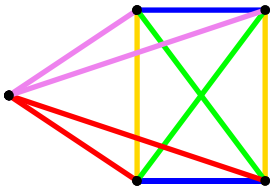 $n = 5$
- k ... num. of edges assigned to every color
    $k = 2$
- rainbow cycle of length at most
    $\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{5}{2} \right\rceil = 3$

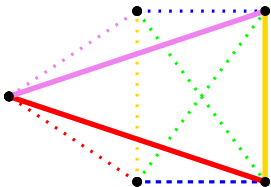# Example

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Thank you

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Thank you for your attention!