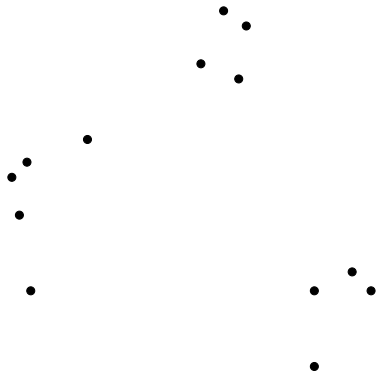
Hardness of Approximating k-Diameter

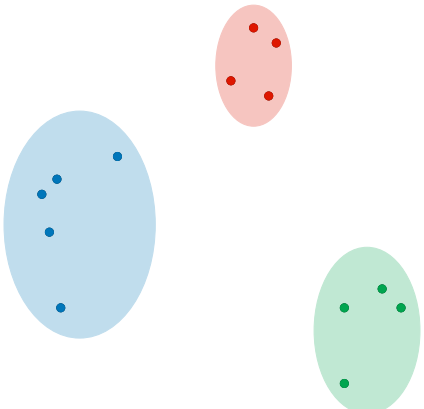
Kyrylo Karlov Ashwin Padaki Styopa Zharkov

Mentors: Karthik C.S. and Henry Fleischmann

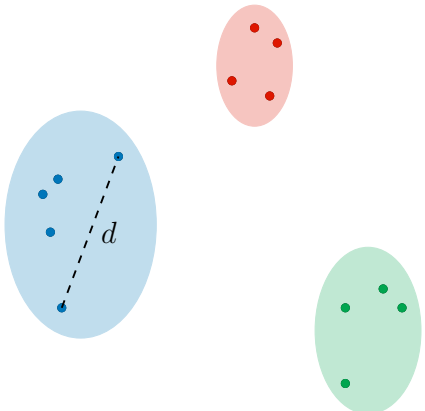
Clustering



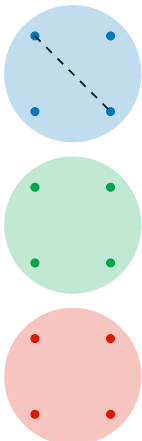
Clustering

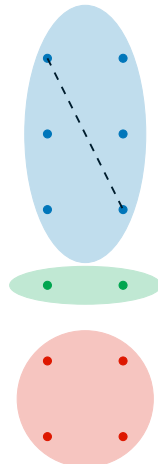


Clustering



Approximate Clustering





Results

Metric	# of Clusters	Hardness	Best Algorithm
$\ell_\infty;\ \ell_0/\ell_1$	unbounded	2	2
ℓ_2	unbounded	1.97	2
ℓ_∞	constant (≥ 3)	2	2
ℓ_0/ℓ_1	constant (≥ 3)	1.5	2
ℓ_2	constant (≥ 3)	1.304	1.414

$$1.304 \approx \sqrt{1+\sqrt{1/2}} - 0.002$$

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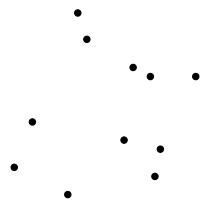
ℓ_2 : “Eyeball” Algorithm ($\sqrt{2}$-approximation)



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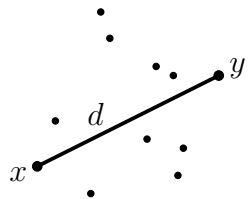

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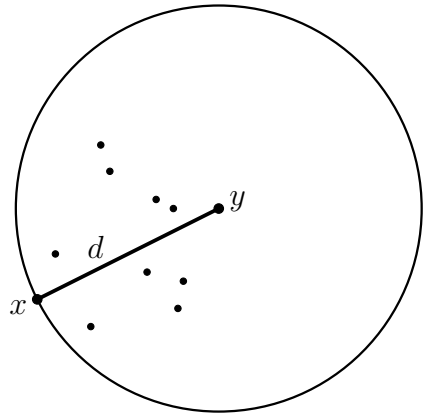

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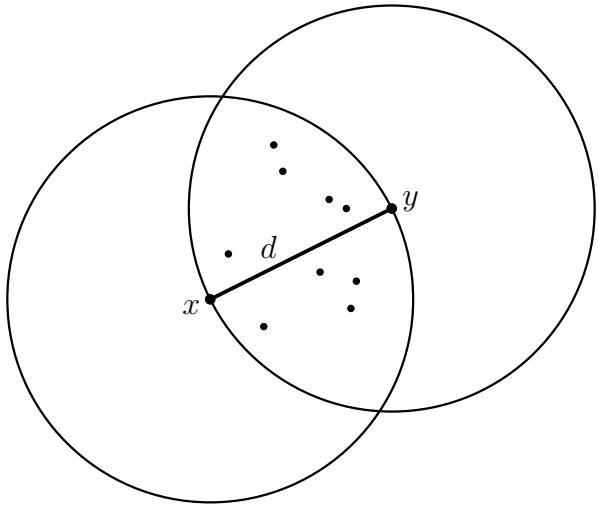

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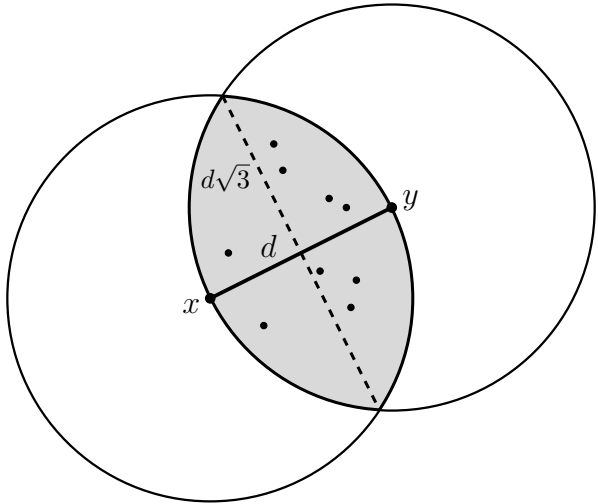

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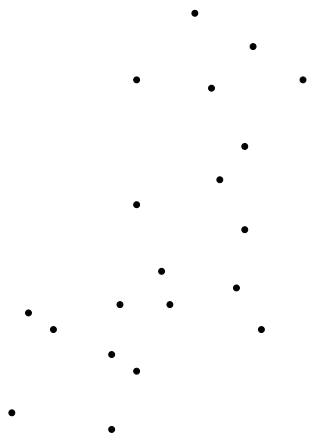

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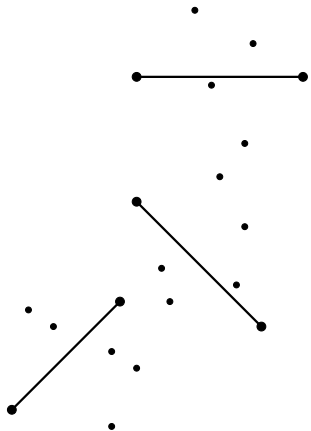

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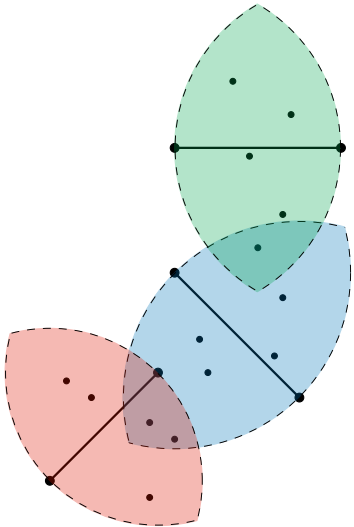

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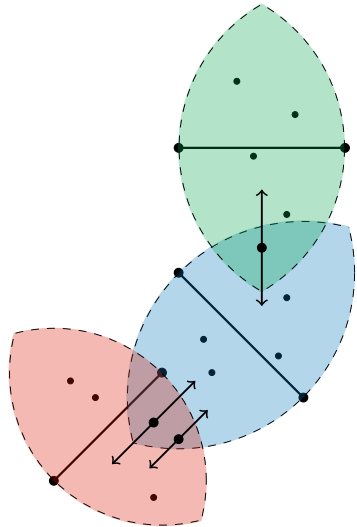

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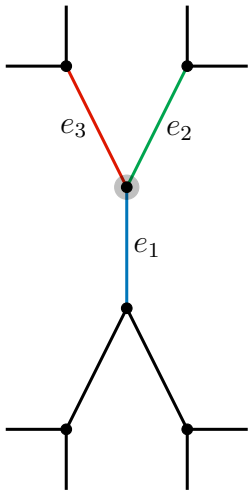
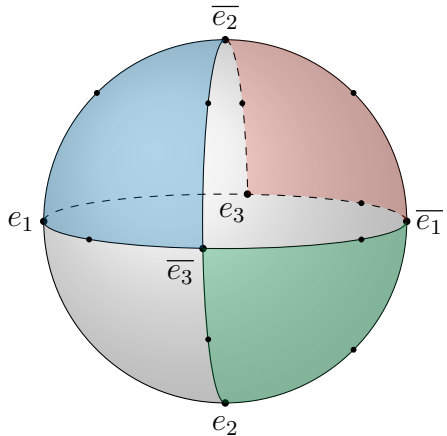
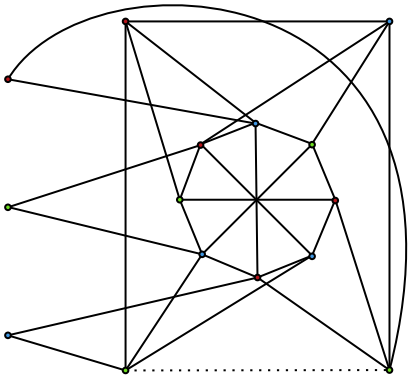
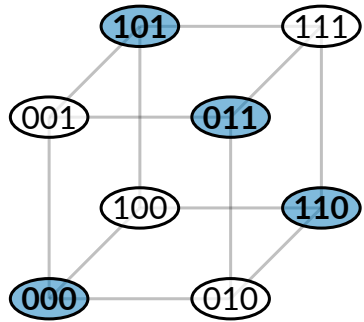
ℓ_2 : Edge Coloring Reduction (1.304-hardness)



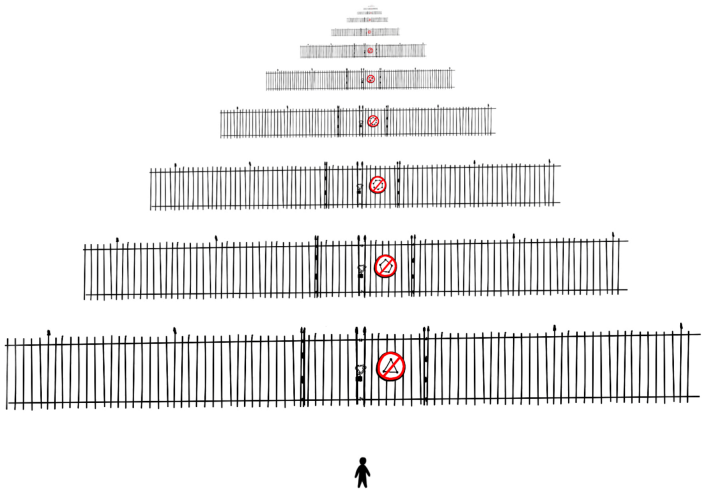


ℓ_0 : Vertex Coloring Reduction (1.5-hardness)





Barriers



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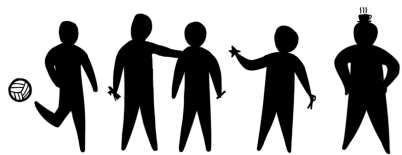

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