FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

Tetrises and Graph Coloring

(joke included)

Aneta Štastná, Ondřej Šplíchal
Mentor: Periklis Papakonstantinou

Motivation

- Suppose that, in a university department, there are *n* committees, each consisting of *n* faculty members, and that all committees meet in the same room, which has *n* chairs.

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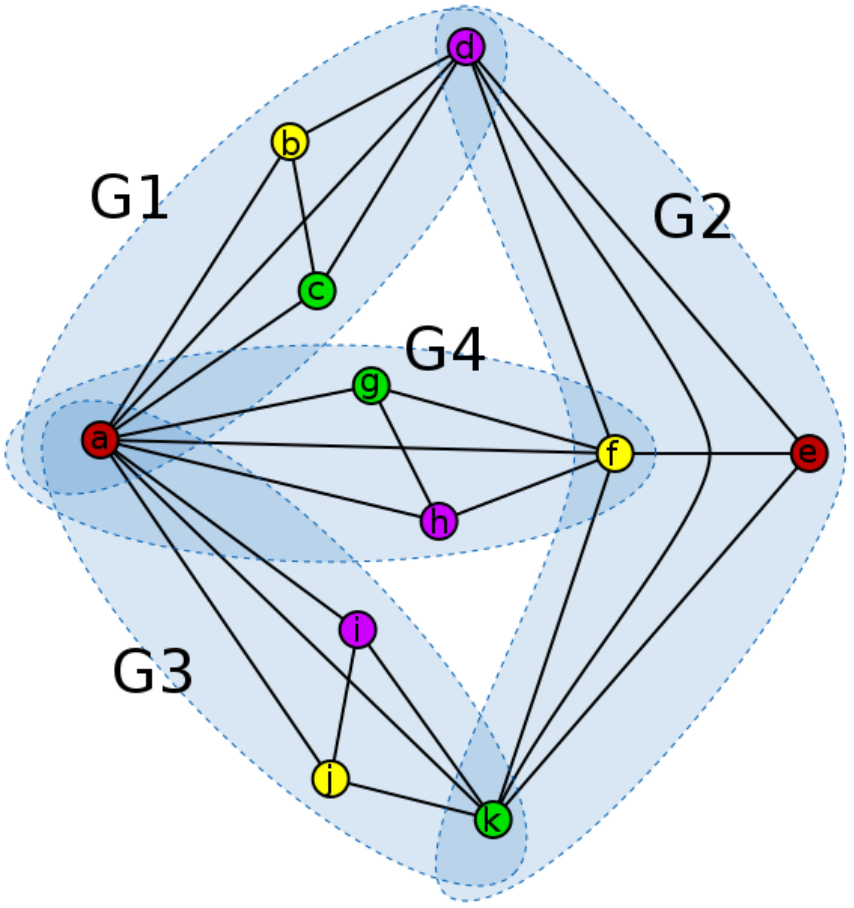
- Suppose that, in a university department, there are *n* committees, each consisting of *n* faculty members, and that all committees meet in the same room, which has *n* chairs.
- Suppose also that at most one person belongs to the intersection of any two committees.
- Is it possible to assign the committee members to chairs in such a way that each member sits in the same chair for all the different committees to which he or she belongs?

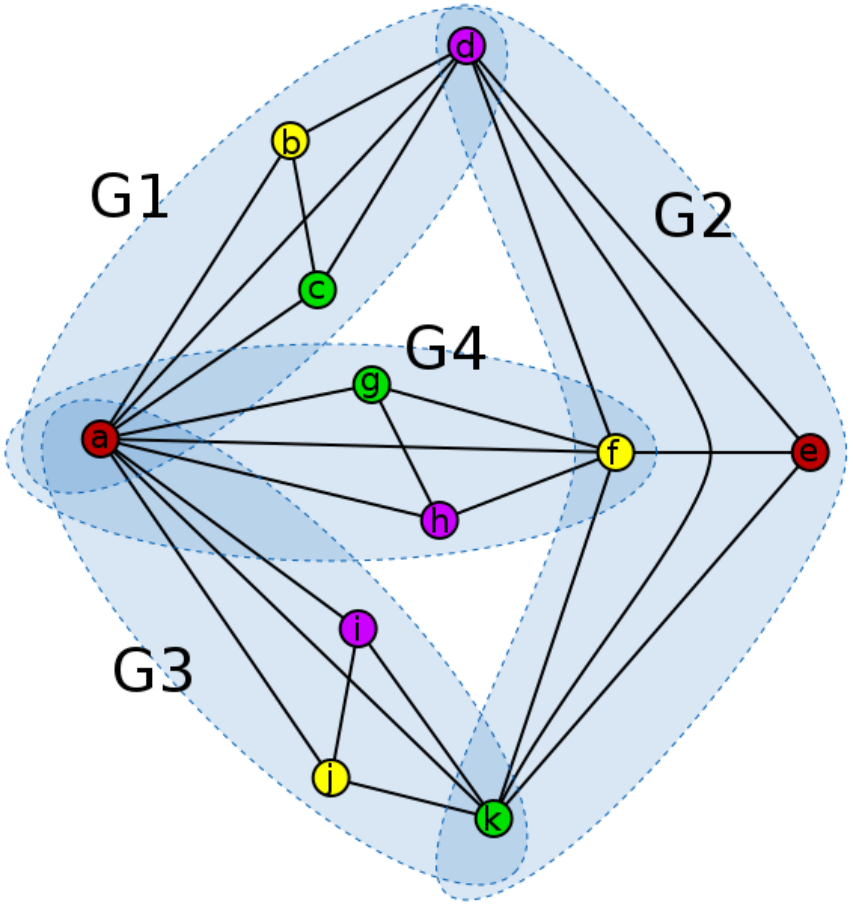
Erdős–Faber–Lovász conjecture

- If *n* complete graphs, each having exactly *n* vertices, have the property that every pair of complete graphs has at most one shared vertex, then the union of the graphs can be colored with *n* colors.



Tetrises

New Perspective on EFL



G1	G2	G3	G4

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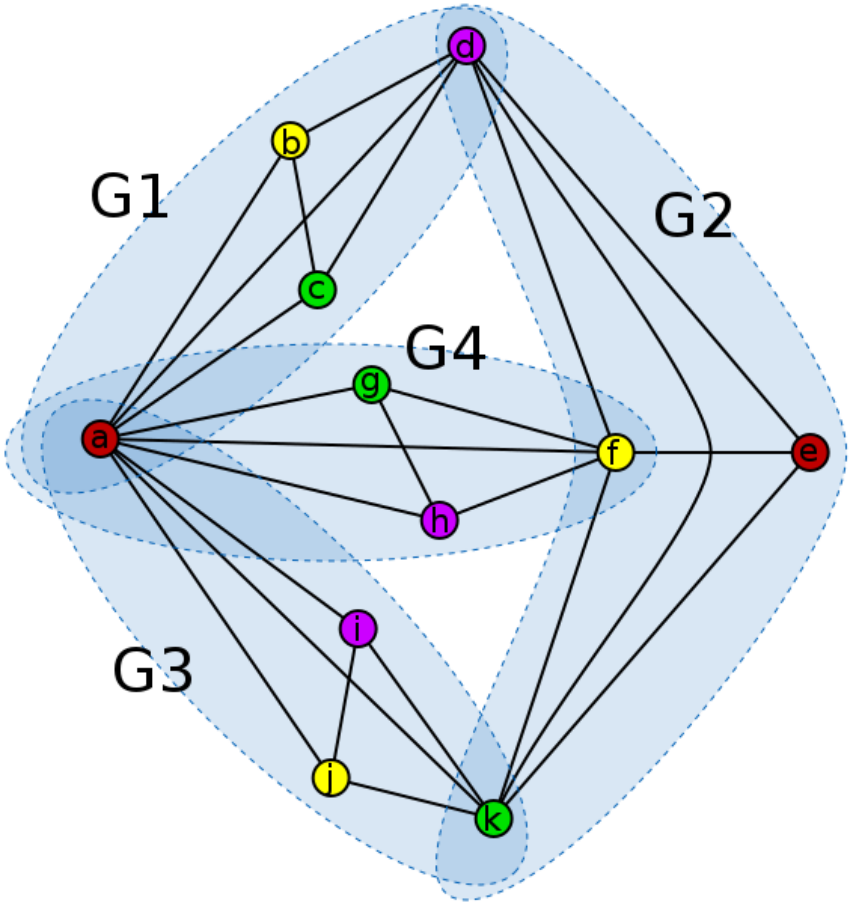

G1	G2	G3	G4
a		a	a

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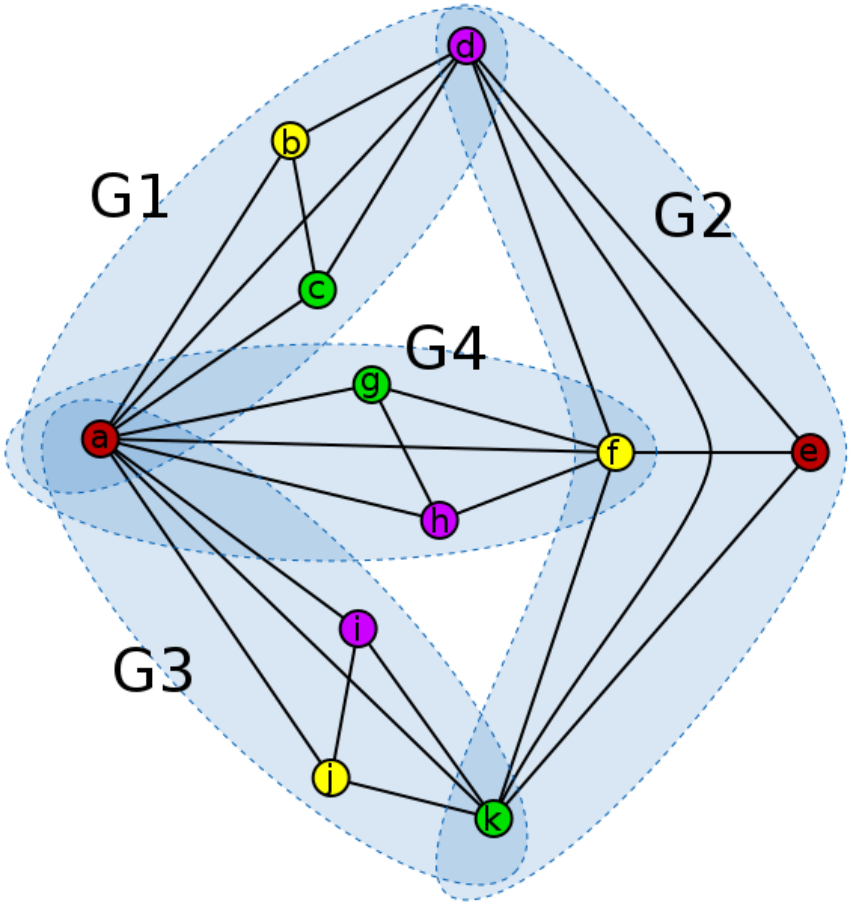

G1	G2	G3	G4
b			
a		a	a

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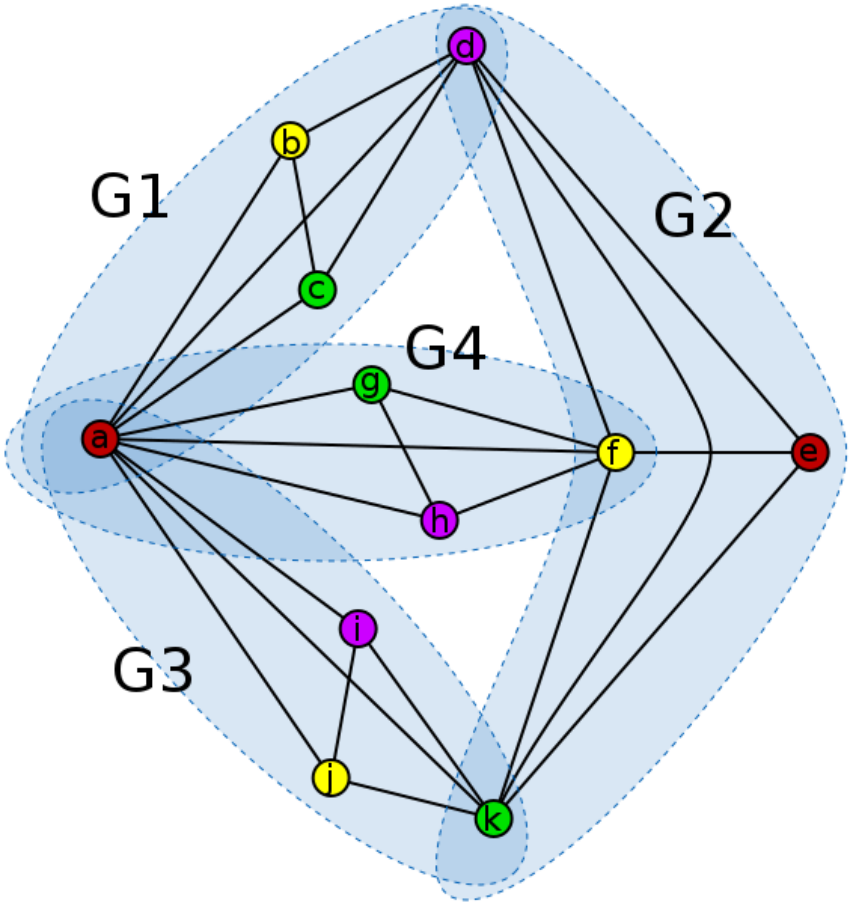

G1	G2	G3	G4
c			
b			
a		a	a

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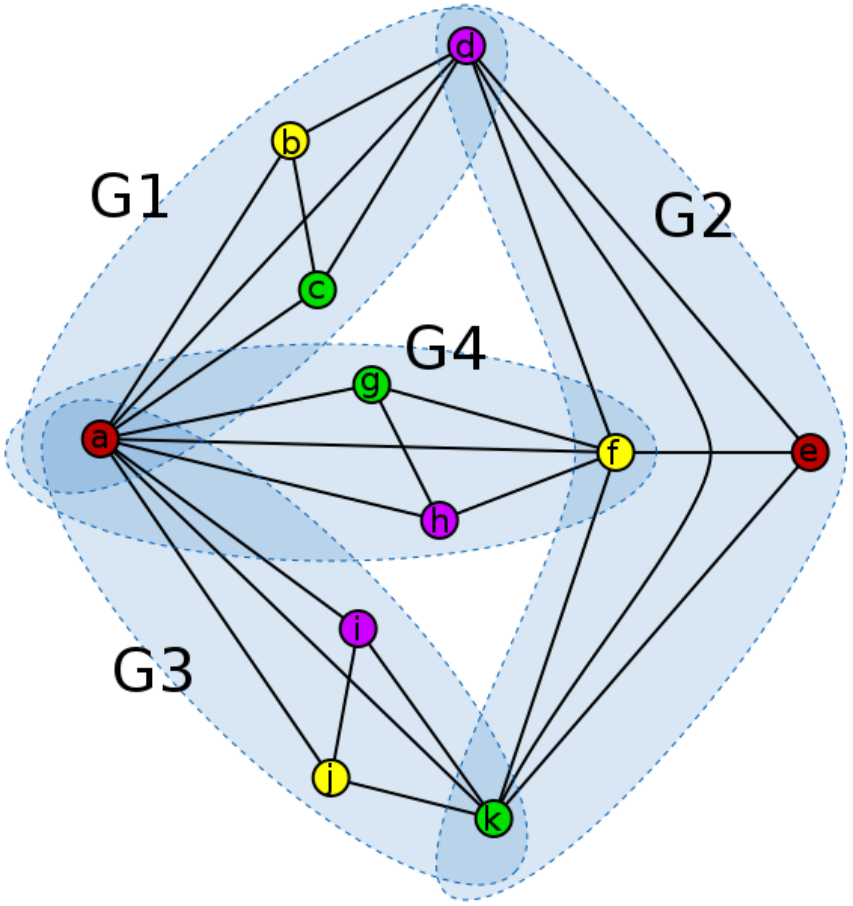

G1	G2	G3	G4
d	d		
c			
b			
a		a	a

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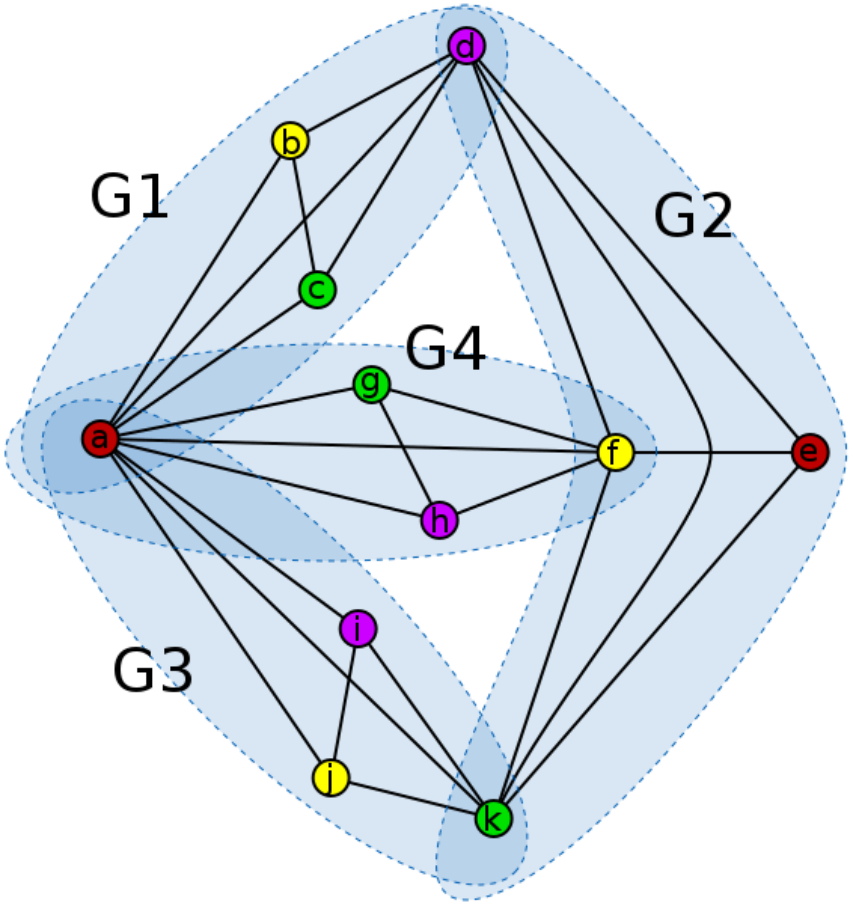

G1	G2	G3	G4
	e		
d	d		
c			
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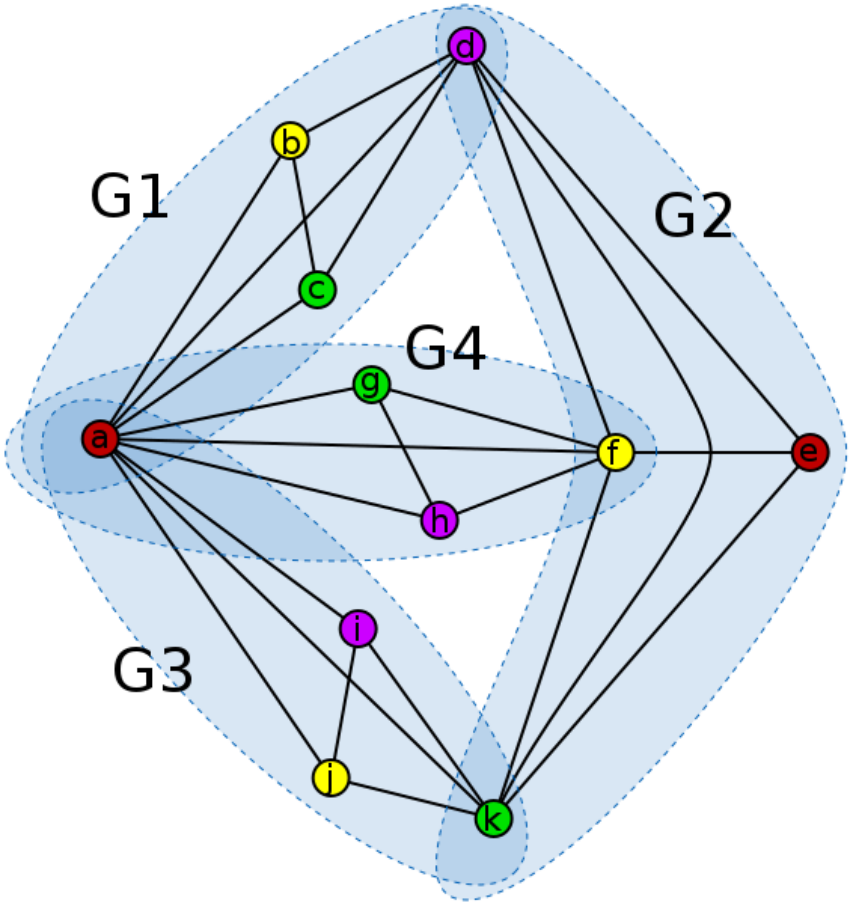

G1	G2	G3	G4
	f		f
	e		
d	d		
c			
b			
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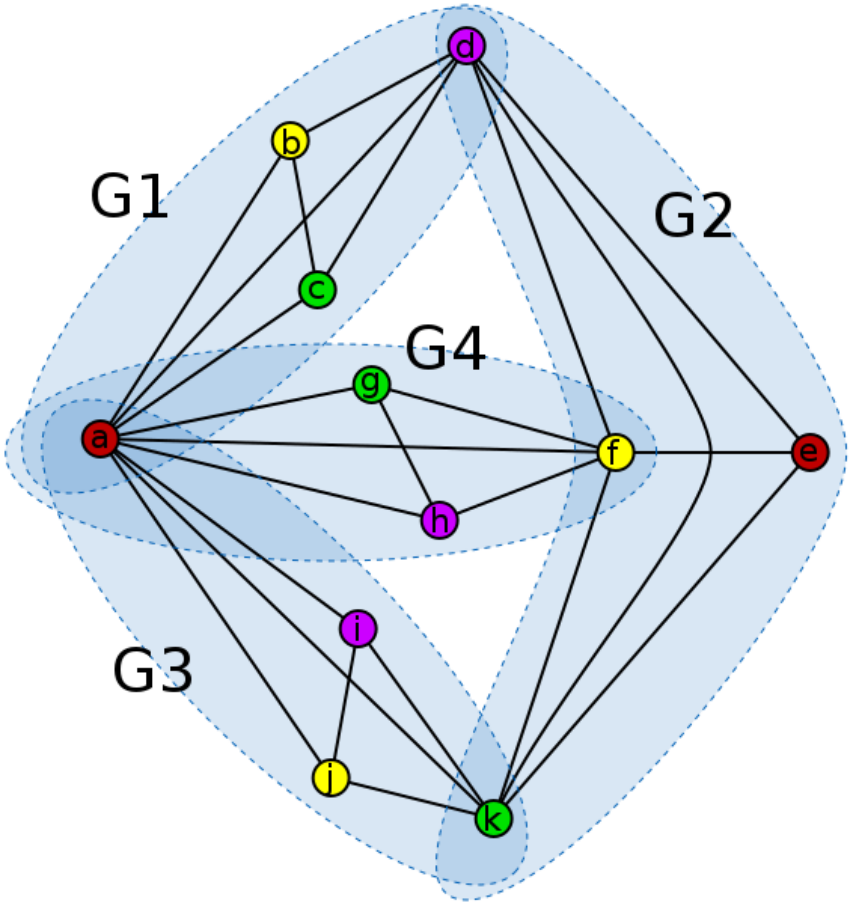

G1	G2	G3	G4
			g
	f		f
	e		
d	d		
c			
b			
a		a	a

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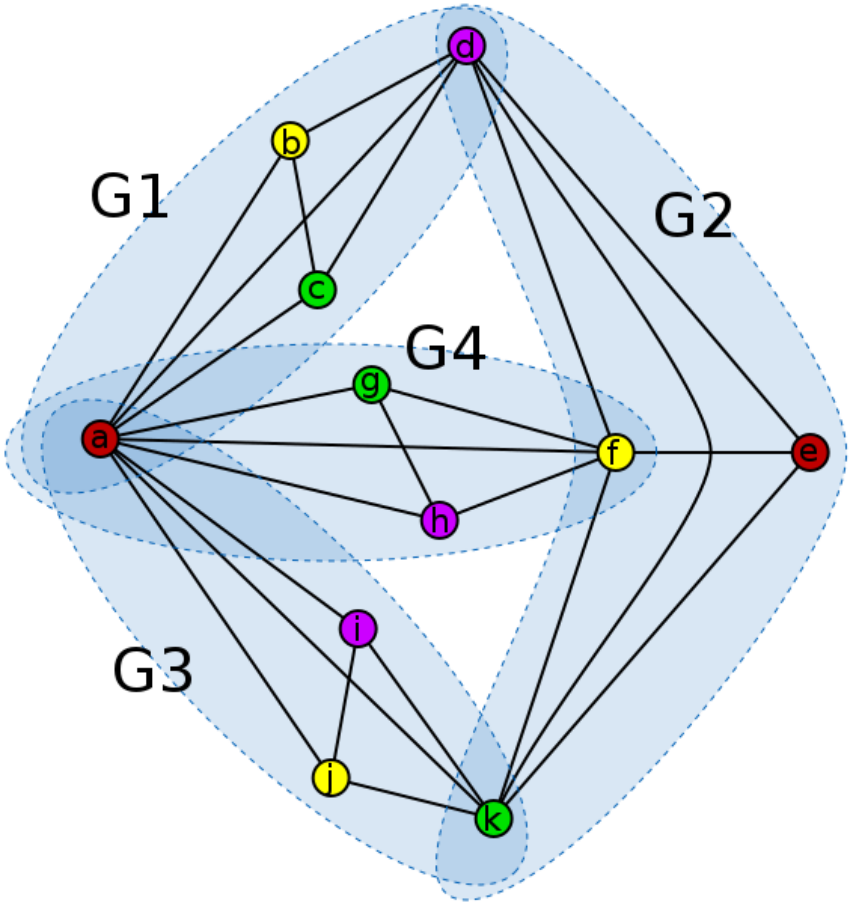

G1	G2	G3	G4
			h
			g
	f		f
	e		
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c			
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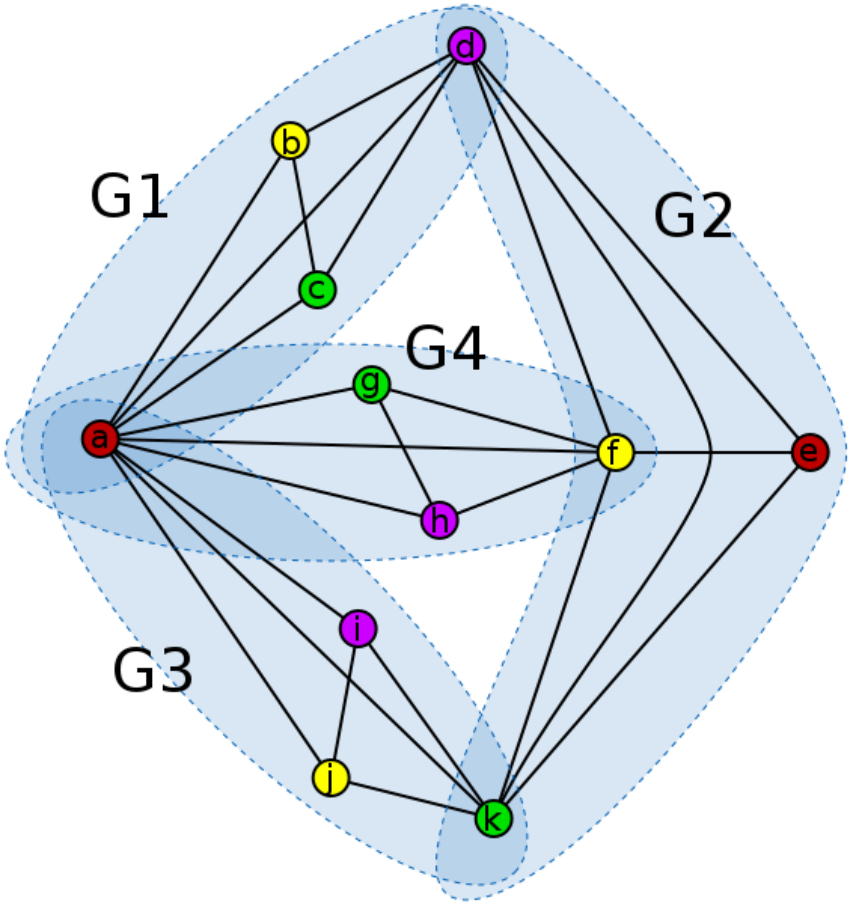

G1	G2	G3	G4
		i	
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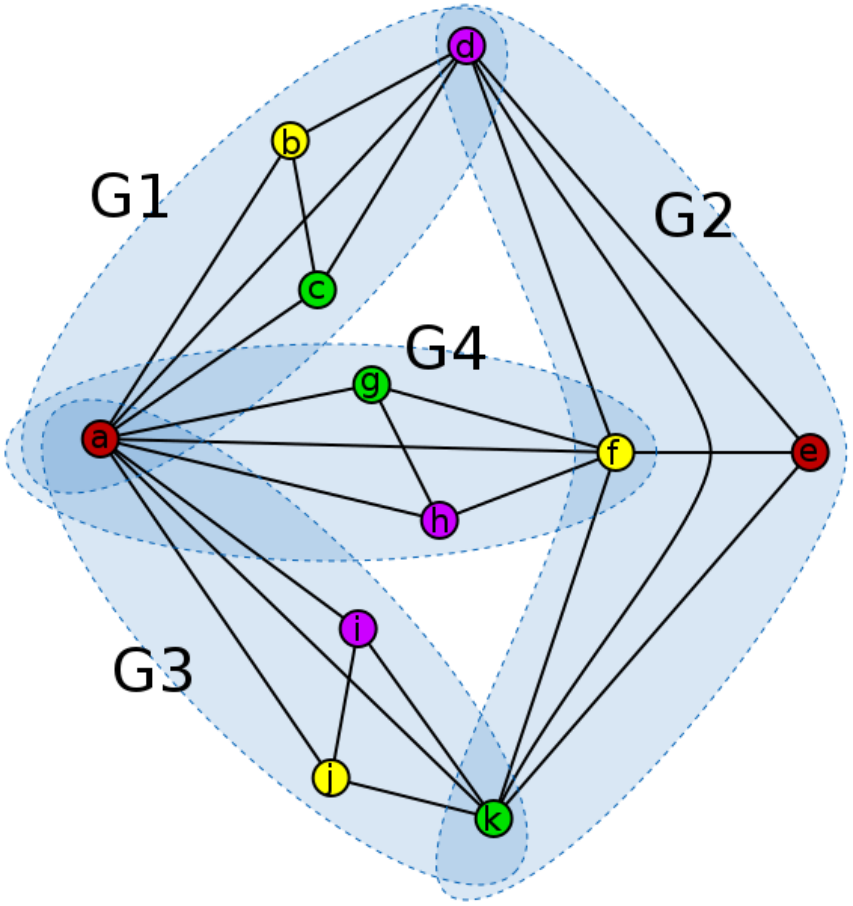

G1	G2	G3	G4
		j	
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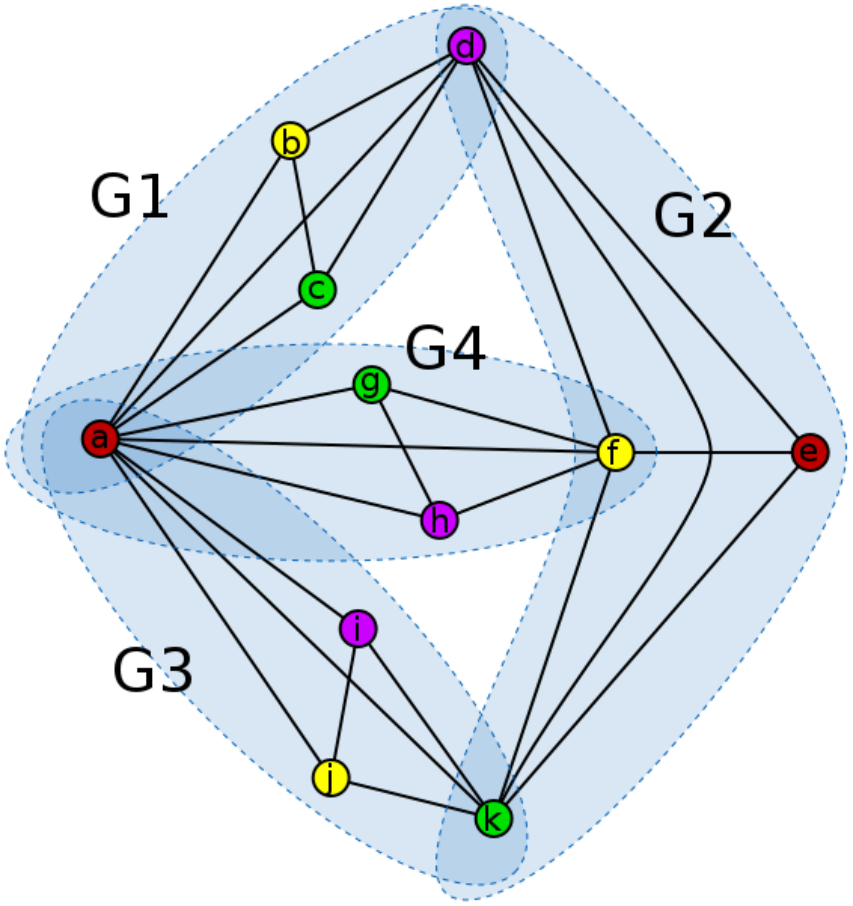

G1	G2	G3	G4
	k	k	
		j	
		i	
			h
			g
	f		f
	e		
d	d		
c			
b			
a		a	a

Tetrises

New Perspective on EFL

- Defining property:

  Every two bricks (rows) have at most one filled tile in common.

G1	G2	G3	G4
	k	k	
		j	
		i	
			h
			g
	f		f
	e		
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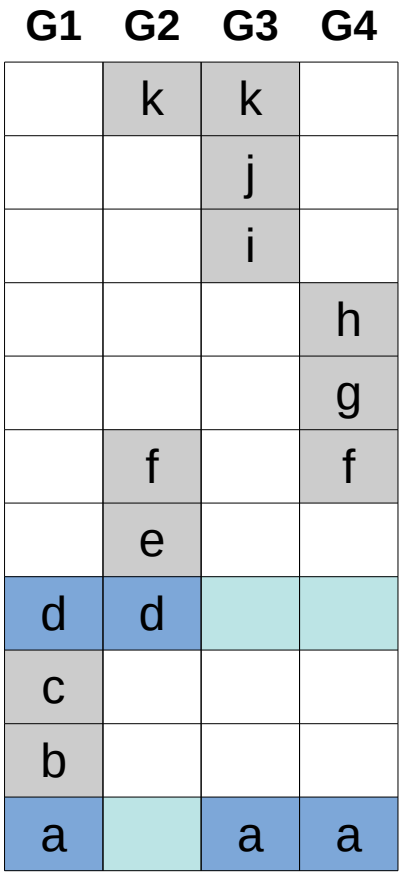

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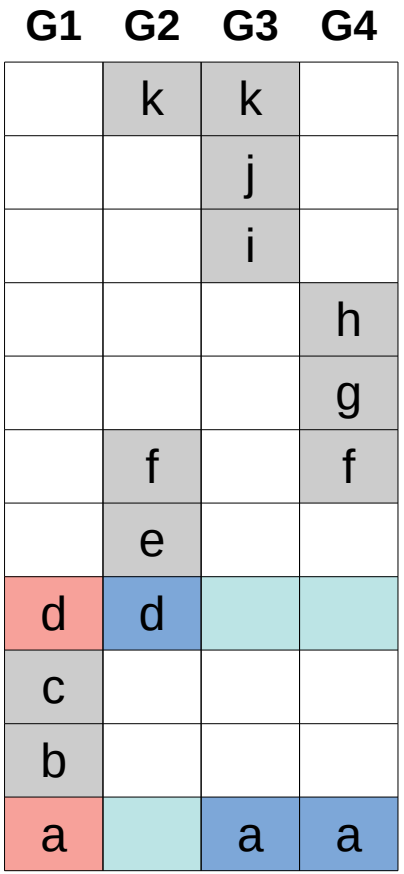

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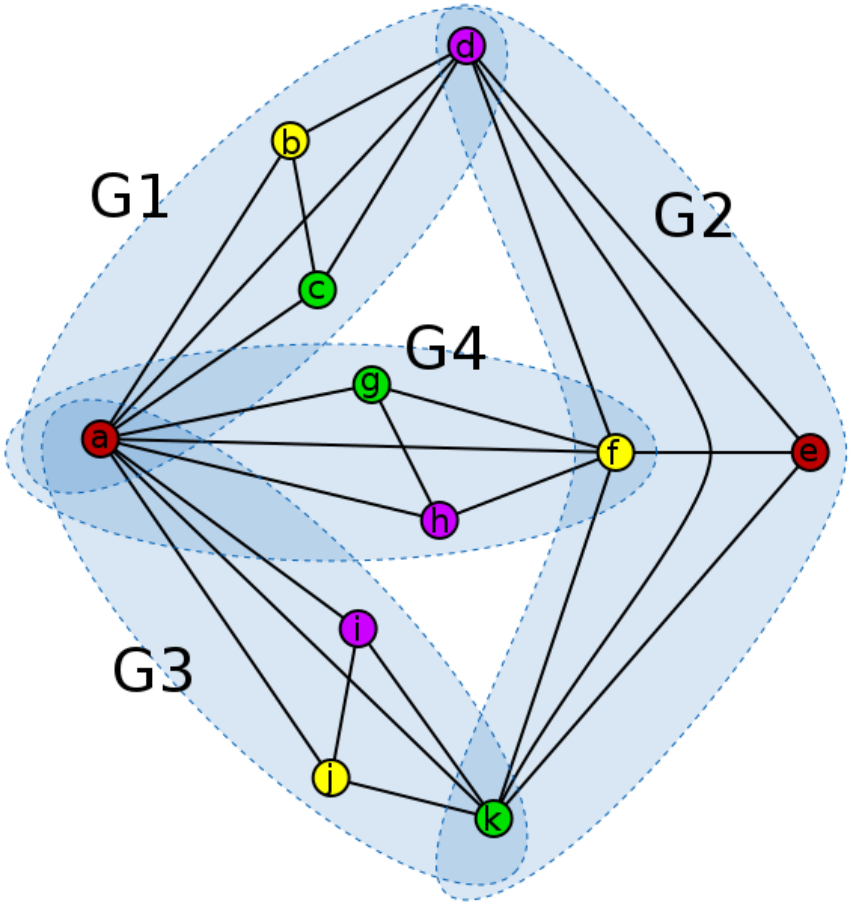

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	k	k	
		j	
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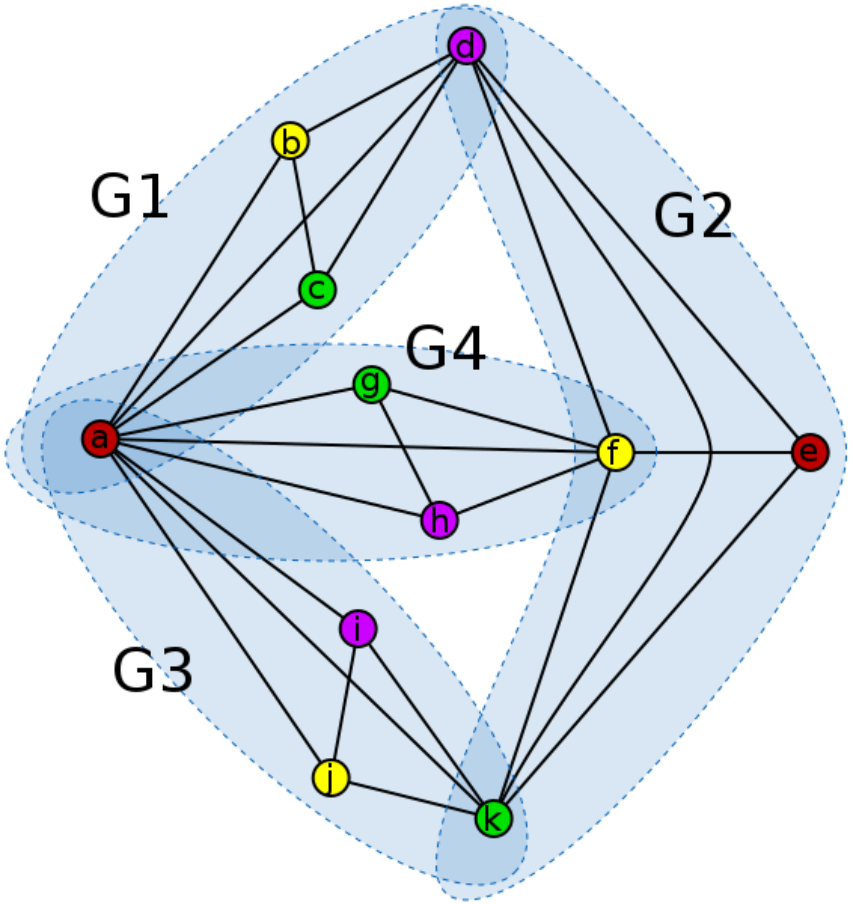

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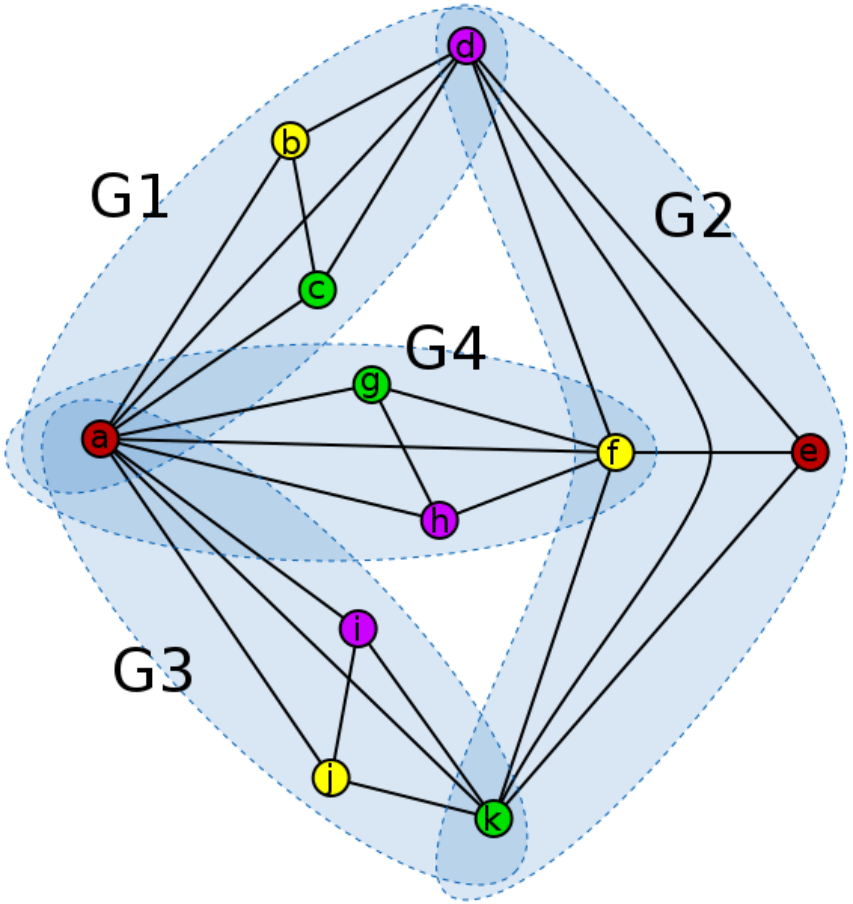

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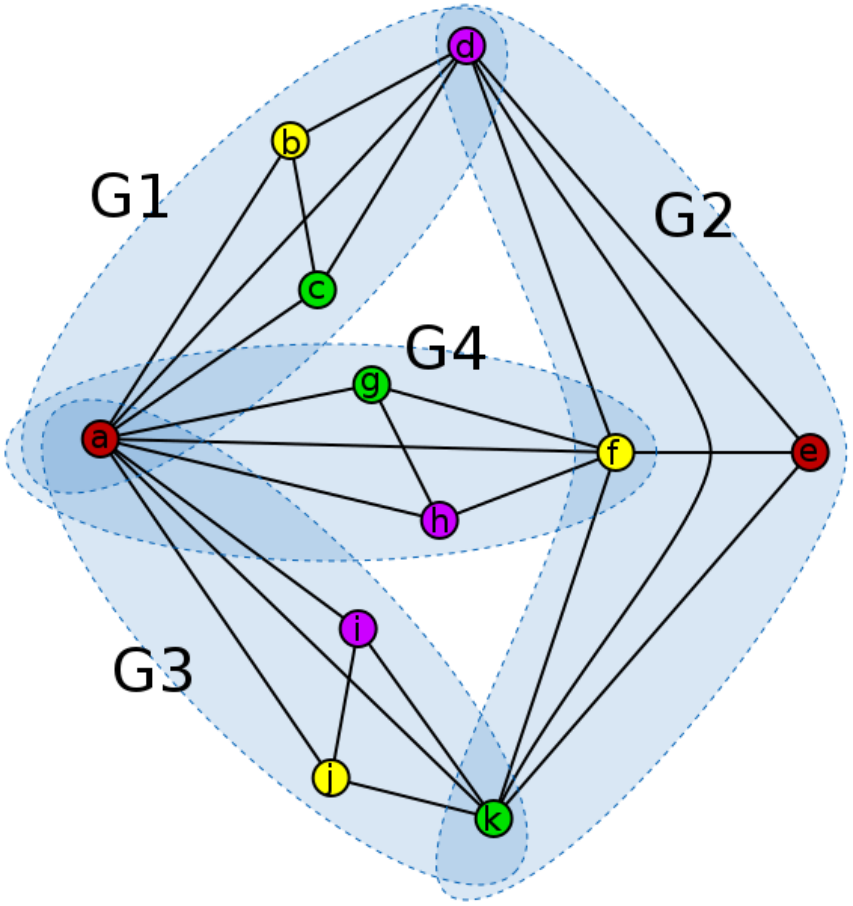

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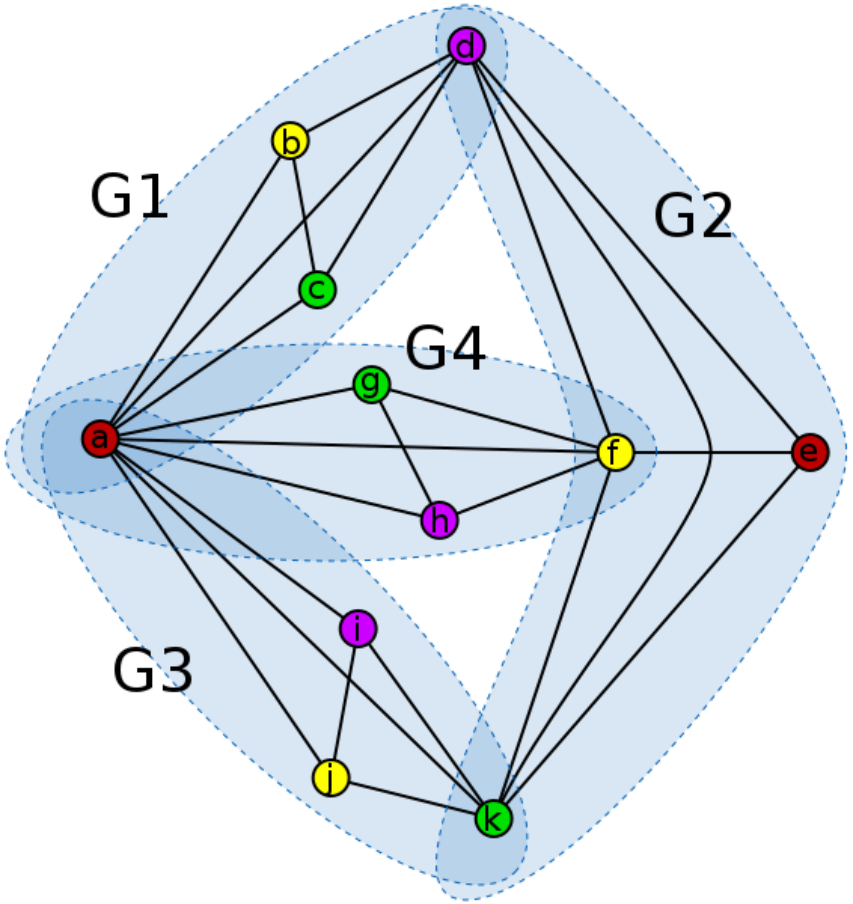

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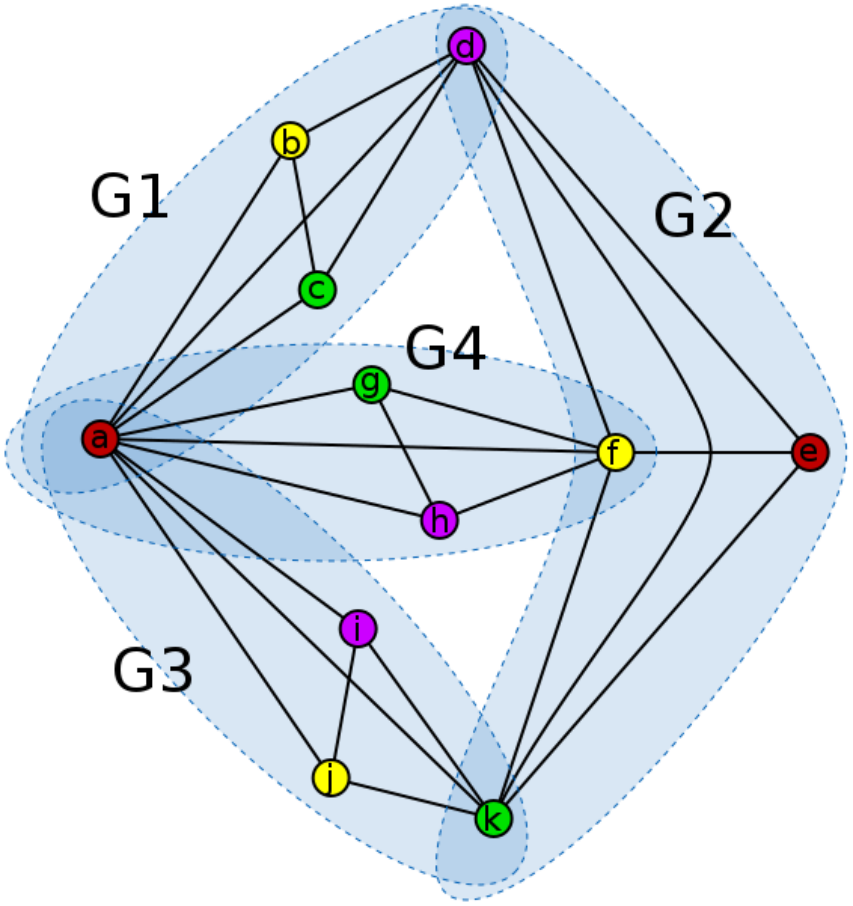

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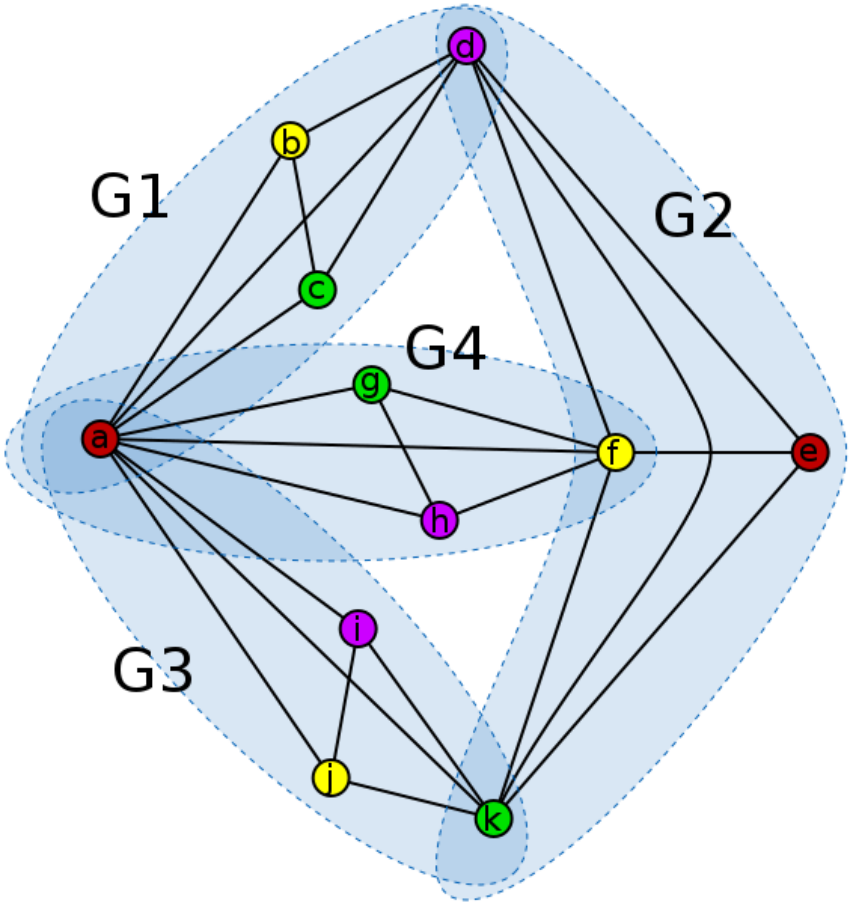

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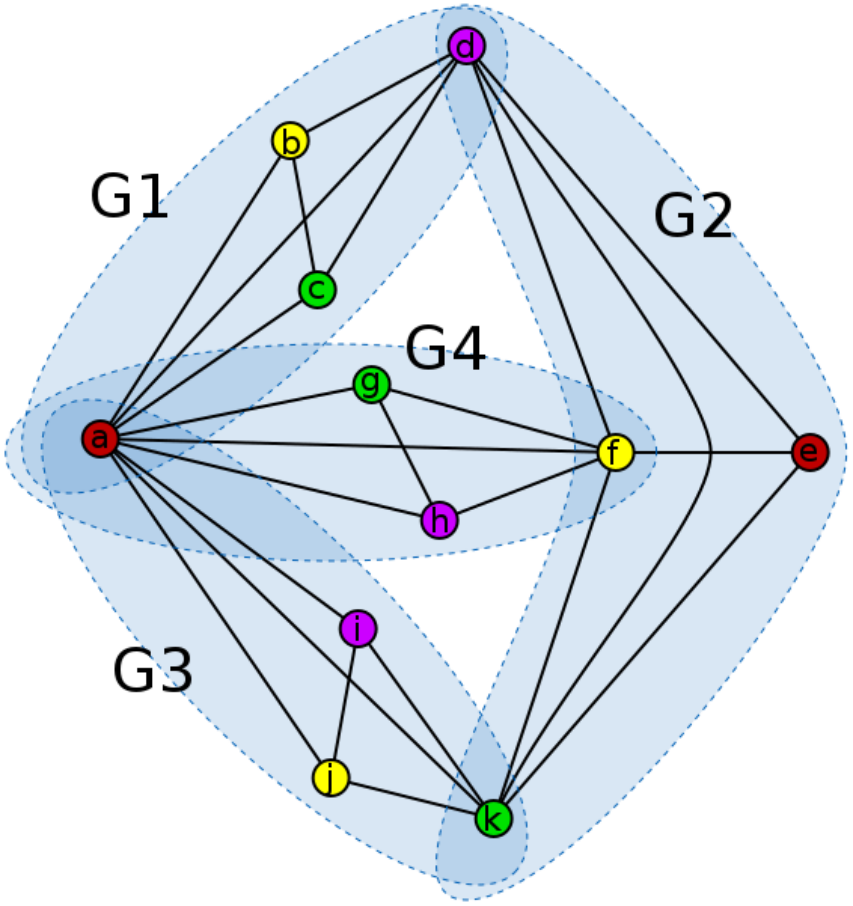

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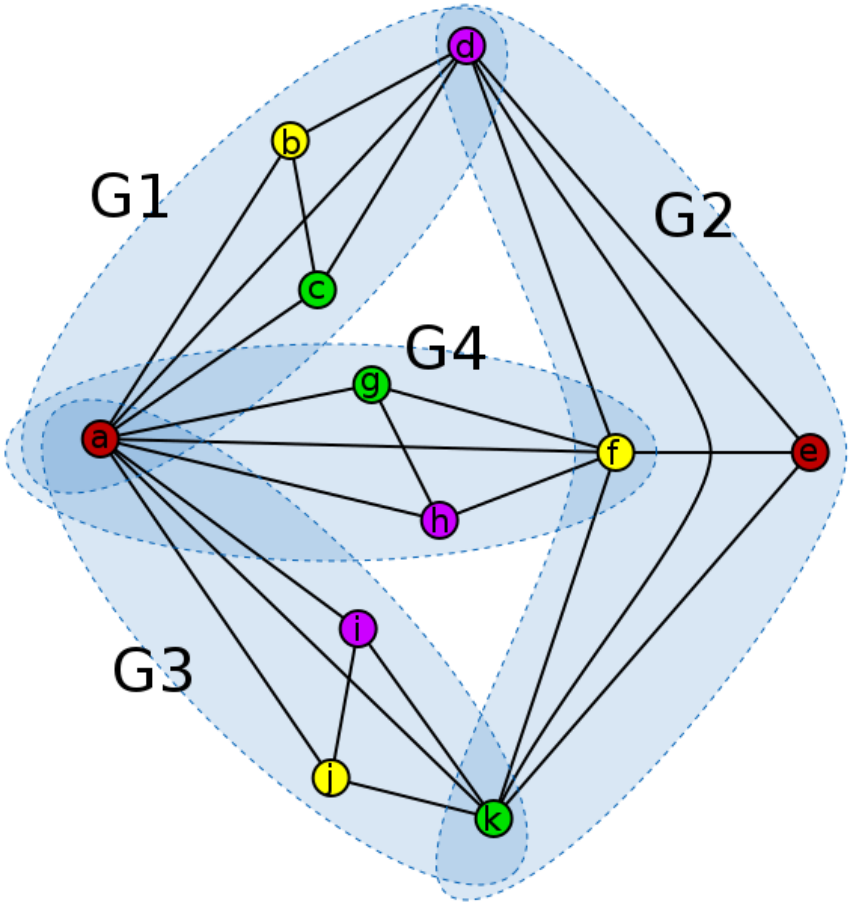

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c			g
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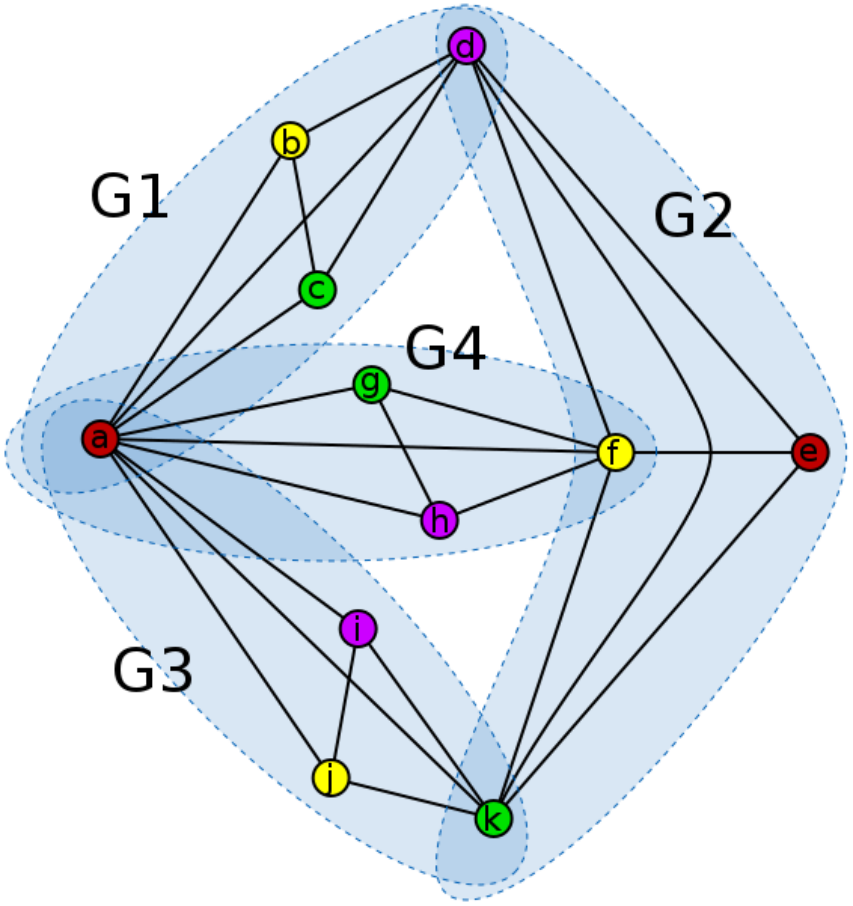

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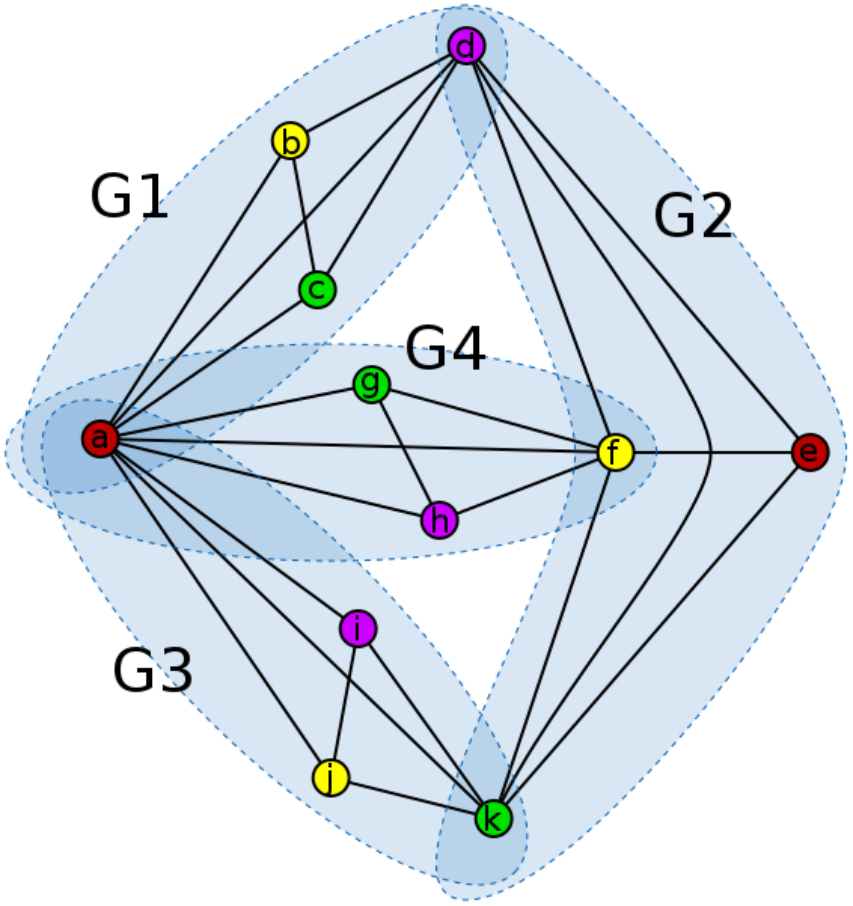

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		i	
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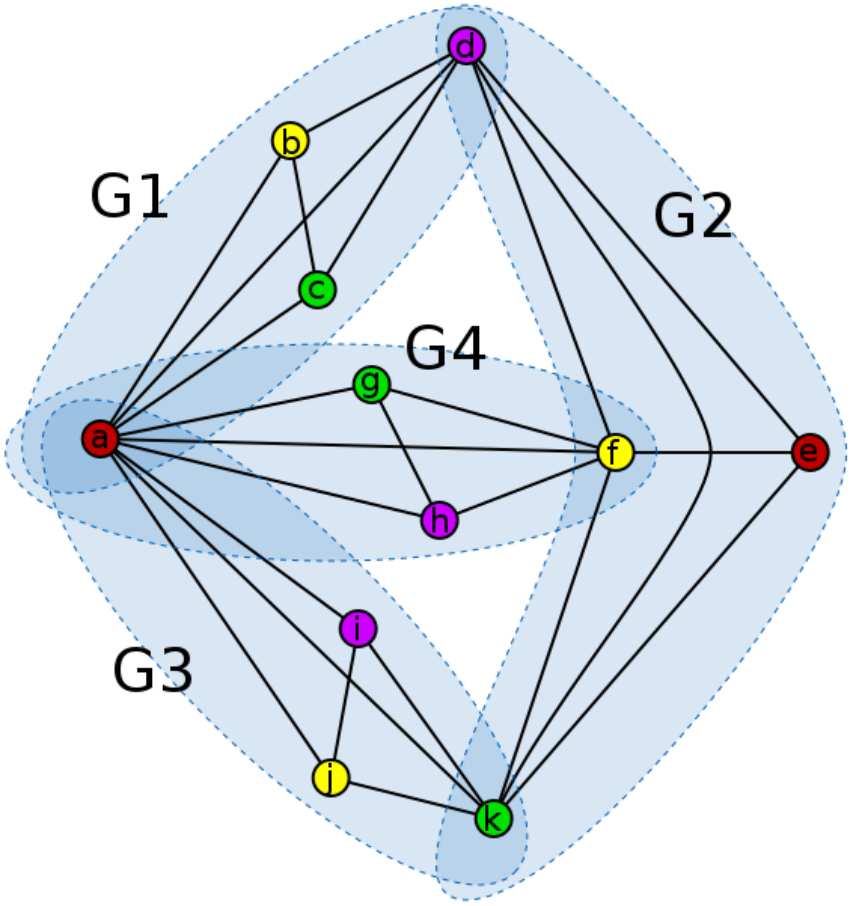

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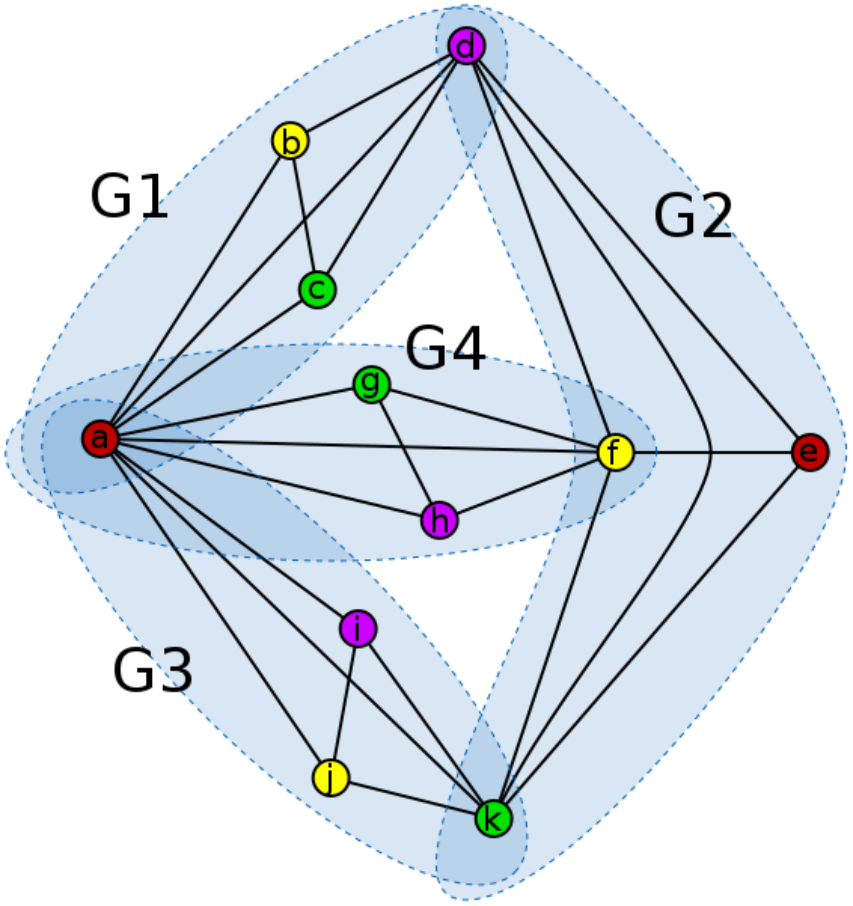

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			h
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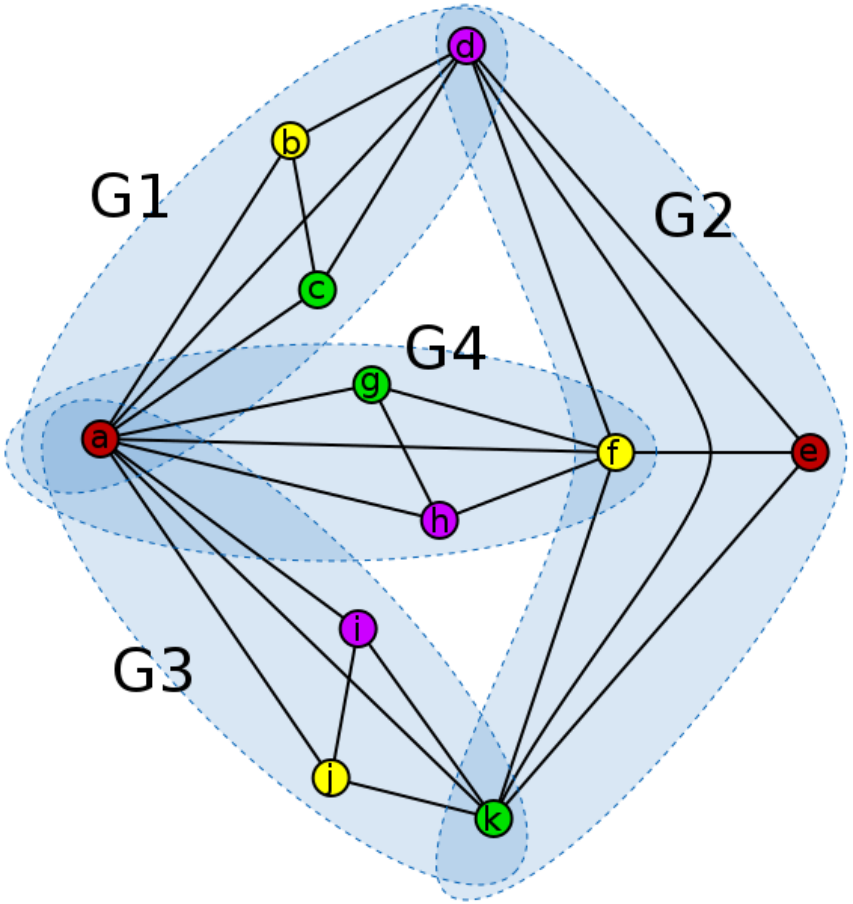

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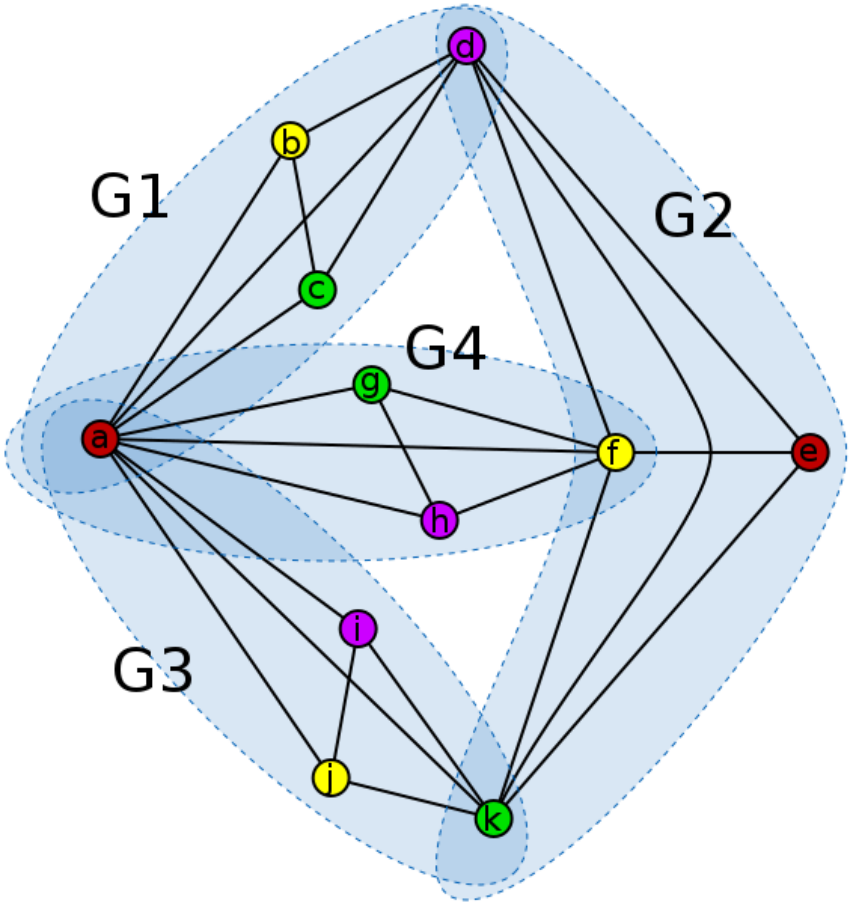

G1	G2	G3	G4
d	d	i	h
c	k	k	g
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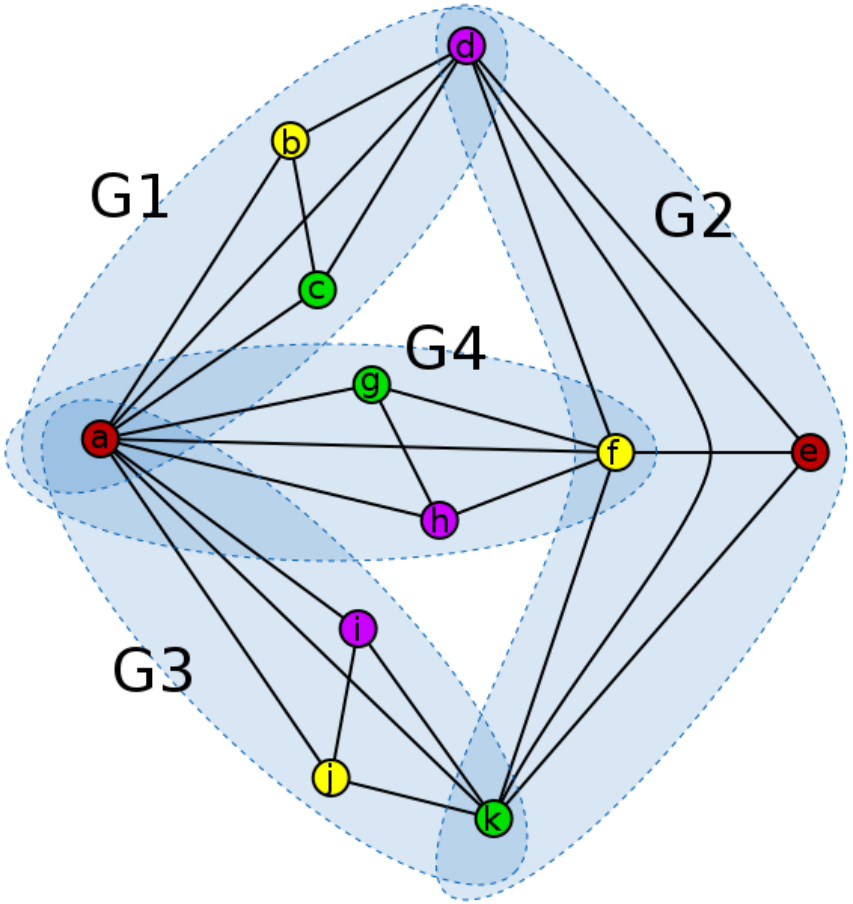

G1	G2	G3	G4
d	d	i	h
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- Learn about known results on EFL

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  - holds for $n < 13$
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- Learn about known results on EFL
- Reformulate and improve these results in terms of tetrises:
  - holds for $n < 13$
  - holds for dense graphs
  - etc.
- Make existing proofs more clear using tetrises
- Discover new things and hidden relations

Joke

Thanks for watching!