Compactness in the Mathematical Universe

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Compactness

Compactness is a phenomenon in mathematics where structures are determined by their local behavior.

Compactness in nature: fractals

Compactness in mathematics

Godel's compactness

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Compactness in mathematics

Godel's compactness

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Topological compactness

A topological space is compact if every open cover of the space has a finite subcover.

Large cardinal axioms

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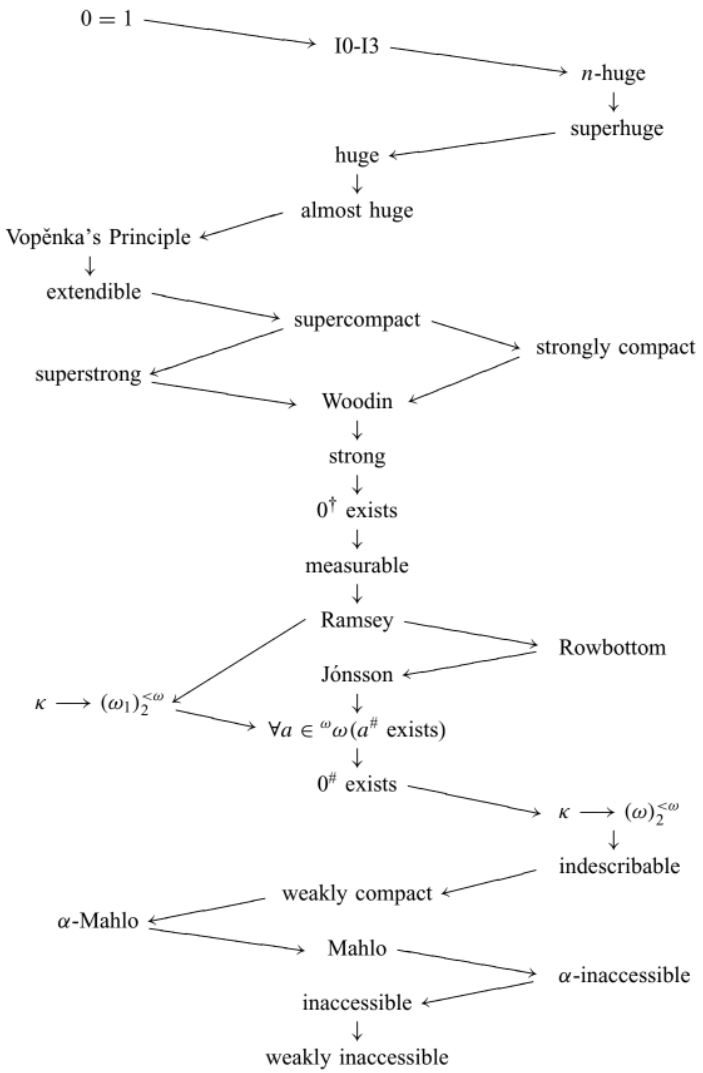
- "ZFC + 'there is an inaccessible cardinal'" has higher consistency strength than ZFC alone.

Large cardinal chart

The arrows indicates direct implications or relative consistency implications, often both.



Compactness with large cardinals

Theorem

Let κ be a weakly compact cardinal. If G is a κ-free group of cardinality less than or equal to κ, then G is free.

REU Goals

- Finding other forms of compactness for large cardinals.

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- Finding other forms of compactness for different algebraic structures.

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