Relationships Between Combinatorial Knot Invariants

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What is a Knot?

- A *knot* is an embedding $S^1 \hookrightarrow S^3 = \mathbb{R}^3 \cup \infty$.
- A *link* is an embedding of a disjoint union $S^1 \cup \cdots \cup S^1 \hookrightarrow S^3$.

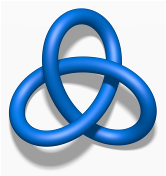

Figure: Trefoil Knot

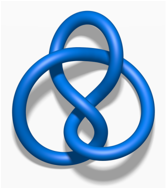

Figure: Figure-Eight Knot

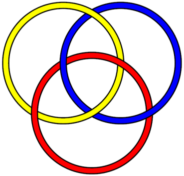

Figure: Borromean Rings: a 3-component link

The source of the images is the Knot Atlas: http://katlas.org/wiki/

Why Knots are Important

- Many things in the real world are knotted - Applications in studying DNA
- Knots are an early case of the embedding problem.
- Knots are very closely related to 3- and 4-dimensional manifolds.

Theorem

(Lickorish, Wallace): Every closed 3-dimensional manifold can be described in terms of some link and an integer associated to each component.

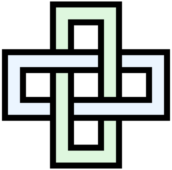

Figure: Solomon's Knot Square: a 2-component link

The source of the image is the Knot Atlas: http://katlas.org/wiki/

What We're Studying

To each knot K we can associate the complex $CFK^\infty(K)$ which contains extensive geometric information about the knot.

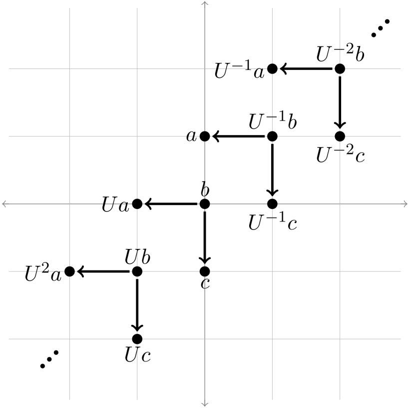

Figure: $CFK^\infty(K)$ for the right-handed trefoil knot

Source of the figure: *A Survey on Heegaard Floer Homology and Concordance* by Jennifer Hom (2017).

The Object We're Looking For

Definition

$\iota_K : CFK^{\infty}(K) \to CFK^{\infty}(K)$

- Contains interesting 4-dimensional data
- Can detect the fact that the figure-eight knot doesn't bound a smooth disk in B^4.

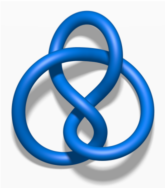

Figure: The figure-eight knot

The source of the image is the Knot Atlas: http://katlas.org/wiki/

What Knots We Will Consider

ι_k has been computed for

- Torus knots
- Alternating knots
- Some pretzel knots (previous REU)

We want to compute ι_k for

- (1,1)-knots (for which ι_k hasn't been computed)

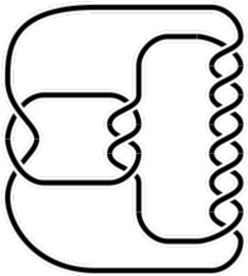

Figure: The pretzel knot $P(-2, 3, 7)$

The source of the image is: https://wikipedia.org/wiki/(-2,3,7)_pretzel_knot

(1, 1)-knots

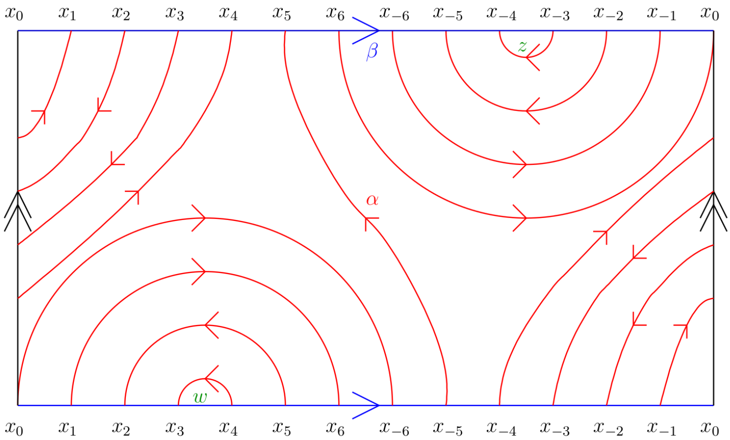

Figure: The knot 10_{161} represented by the 4-tuple $(6, 4, -3, -1)$

Figure source: *Geometry of (1, 1)-Knots and Knot Floer Homology* by Racz

Goals

- Compute ι_K for the 10- and 11-crossing (1,1) knots for which it isn't known
- Understand when a (1,1) diagram gives us enough information to easily compute ι_K

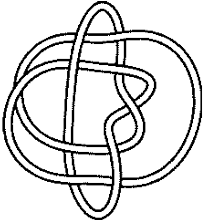

Figure: The knot 10_{161}

The source of the image is the Knot Atlas: http://katlas.org/wiki/

Acknowledgements

- Thank you to our mentors Dr. Hendricks and Karuna Sangam!
- Additional thanks to the Rutgers Math Department for hosting us!
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- Thank you for listening!