Ramsey theory and rado’s numbers

ADRIAN GALICIA
Objective
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- Create an algorithm to compute Rado numbers
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- Implement the algorithm in a web-friendly language (JavaScript, PHP, etc.)
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- Visualizing the computation
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- Implement the algorithm in a web-friendly language (JavaScript, PHP, etc.)
- Visualizing the computation
- Prove the output (Rado numbers) with certificates.
Ramsey Theorem
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- If two colors are used (red & blue), for any positive integers (r,s), there exist a positive integer \( N = R(r,s) \) where a coloring of \( K[N] \) will give \( K[r] \) red or \( K[s] \) blue.
Ramsey's Theorem

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- If two colors are used (red & blue), for any positive integers \((r, s)\), there exist a positive integer \(N = R(r, s)\) where a coloring of \(K[N]\) will give \(K[r]\) red or \(K[s]\) blue.
- The \(R(r, s)\) is smallest integer \(N\) for which the theorem holds.
Ramsey’s Theorem example
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Ramsey’s theorem example
Rado number
Rado number

- Given some equation “E” such as $x+y = z$ and some number of colors “r” ($r=2$, red and blue in this case), find the $N$ so that there is no way to color 1 through $N$ without a solution to the equation $E$ of the same color.
Given some equation “E” such as \( x+y = z \) and some number of colors “r” (\( r=2 \), red and blue in this case), find the \( N \) so that there is no way to color 1 through \( N \) without a solution to the equation \( E \) of the same color.

This is analogous to \( N=R(r,s) \), like \( R(3,3)=6 \). As showed in the graph, \( K[5] \) is the smallest number you can color and avoid monochromatic \( K[3] \).
Rado number example

\[ x + y = z \]
Rado number example

- $N = 1$

1 1

$x + y = z$
### Rado number example

- \( N = 1 \)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ x + y = z \]
Rado number example

<table>
<thead>
<tr>
<th>( N = 1 )</th>
<th>( \begin{array}{cc} 1 &amp; 1 \end{array} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 2 )</td>
<td>( \begin{array}{ccc} 1 &amp; 2 &amp; 1 \end{array} ) ( \begin{array}{cc} 2 &amp; 1 \end{array} ) ( \begin{array}{cc} 1 &amp; 2 \end{array} ) ( \begin{array}{cc} 1 &amp; 2 \end{array} )</td>
</tr>
</tbody>
</table>

\( 1+1 = 2 \) \( 1+1 = 2 \)
Rado number example

- $N = 1$
  - $1 + 1 = 2$
  - $1$  $1$

- $N = 2$
  - $12$  $1$  $2$
  - $1$  $2$
  - $12$
Rado number example

- $N = 1$
  \[
  \begin{array}{ccc}
  1 & 1 \\
  \end{array}
  \]

- $N = 2$
  \[
  \begin{array}{ccc}
  1 & 2 & 1 \\
  \end{array}
  \]

- $N = 3$
  \[
  \begin{array}{ccc}
  1 & 2 & 3 \\
  \end{array}
  \]

$x + y = z$

$1 + 1 = 2$

$1 + 1 = 2$

$1 + 1 = 2$
Rado number example

- $N = 1$
  - $1 + 1 = 2$
  - $1$
  - $1$

- $N = 2$
  - $1 + 1 = 2$
  - $1$
  - $2$
  - $1$
  - $2$

- $N = 3$
  - $1$
  - $2$
  - $3$
  - $1$
  - $2$
  - $3$
  - $1$
  - $2$
  - $3$
  - $1$
  - $2$
  - $3$

- $N = 4$
  - $1$
  - $2$
  - $3$
  - $4$
  - $1$
  - $2$
  - $3$
  - $4$
  - $1$
  - $2$
  - $3$
  - $4$
  - $1$
  - $2$
  - $3$
  - $4$
Rado number example

- **N = 1**
  
- **N = 2**
  
- **N = 3**
  
- **N = 4**

\[
x + y = z
\]

\[
\begin{array}{c c c c}
1 & 1 & \text{1+1 = 2} & \text{1+1 = 2} \\
1 & 2 & 1 & 2 \\
1 & 2 & 3 & 1 \\
1 & 2 & 3 & 4 \\
\end{array}
\]
Rado number example

- **N = 1**
  - 1+1 = 2
  - 1 1

- **N = 2**
  - 1 2
  - 1 2
  - 1 2 1 2

- **N = 3**
  - 1 2 3
  - 1 2 3
  - 1 2 3
  - 1 2 3

- **N = 4**
  - 1 2 3 4
  - 1 2 3 4
  - 1 2 3 4
  - 1 2 3 4
  - 1 2 3 4

- **N = 5**
  - 1 2 3 4 5
  - 1 2 3 4 5
  - 1 2 3 4 5
  - 1 2 3 4 5
  - 1 2 3 4 5

\[ x + y = z \]
Rado number example

- **N = 1**
  - $1+1 = 2$
  - 

- **N = 2**
  - 

- **N = 3**
  - 

- **N = 4**
  - 

- **N = 5**
  - 

$x+y = z$
Ultimate goal
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- Make a program that gives out the least Rado number given some equation taken from the user input.
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- If possible, it will not only work with 2 colors but with 3 or more colors.
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- If possible, it will not only work with 2 colors but with 3 or more colors.
- It will have a tree stylized output (certificate).
Tree output
Tree output

Ø
Tree output

Ø

1 → 1
Tree output

Ø

1

2

1

2
Tree output
Tree output

Can be omitted. mirror image.
Tree output

Can be omitted. mirror image.
Tree output

Can be omitted. mirror image.
Tree output

Can be omitted. mirror image.
Tree output

Can be omitted.
mirror image.
Can be omitted. mirror image.
Tree output

Can be omitted. mirror image.
Thanks for your attention.