Circle Packing & The Koebe-Andreev-Thurston Theorem

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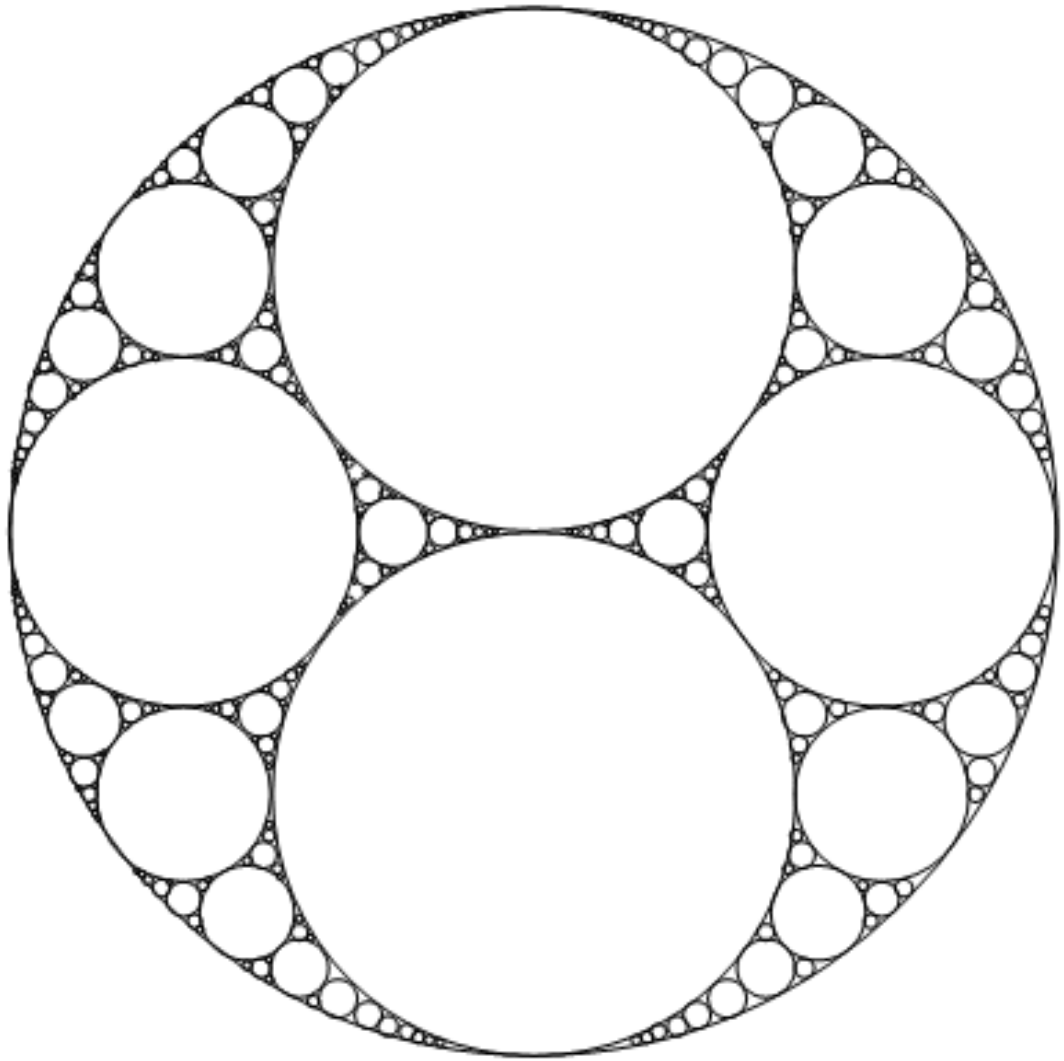
Apollonian Circle Packing



Apollonian Circle Packing

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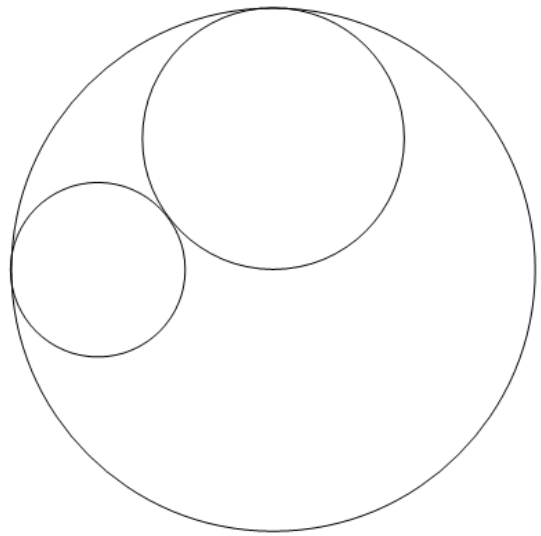
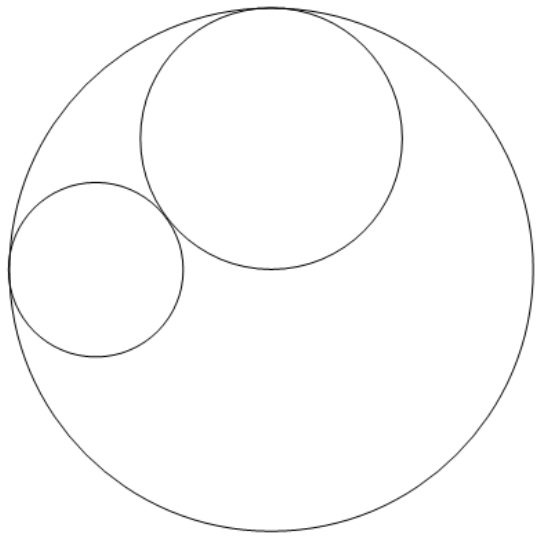


Apollonian Circle Packing

- Start with three mutually tangent circles
- Draw two more circles, each of which is tangent to the original three
  - These come from Apollonius



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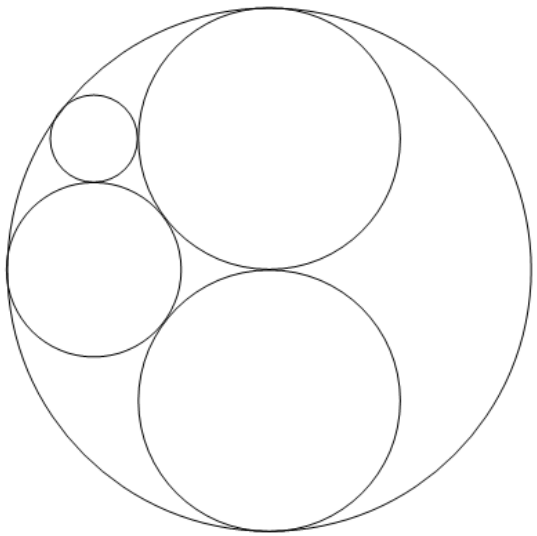
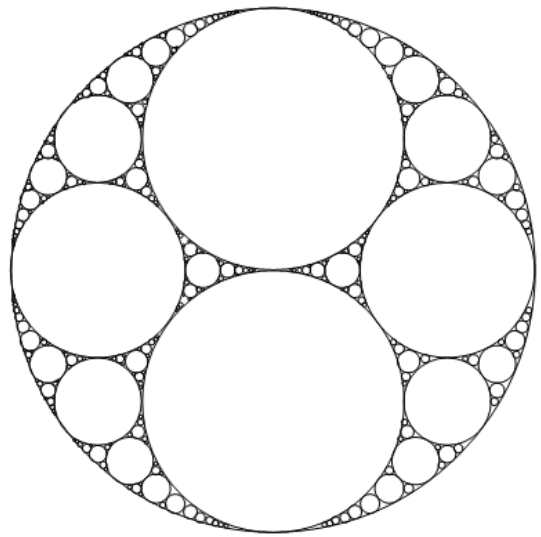


Apollonian Circle Packing

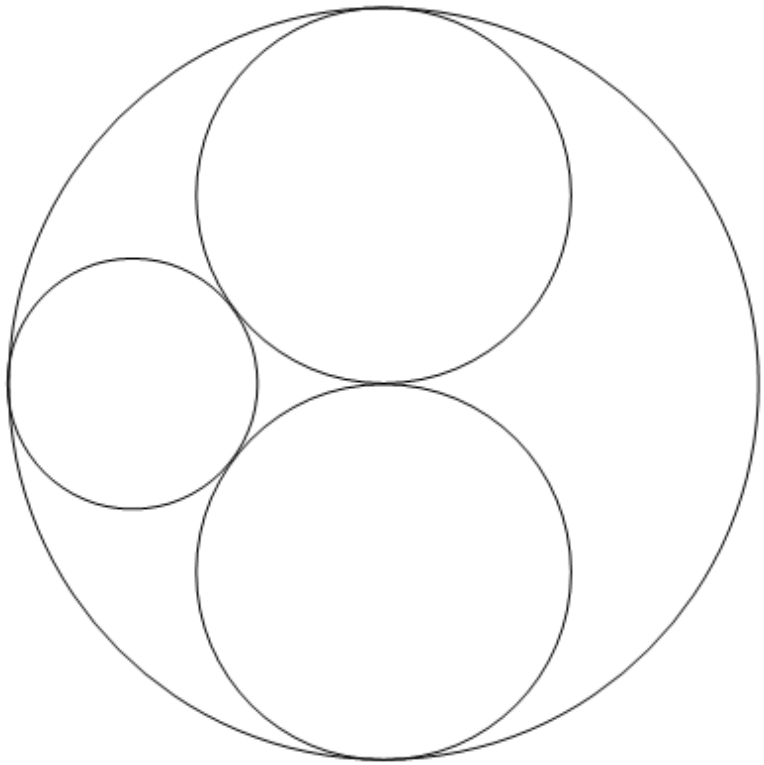
- Start with three mutually tangent circles
- Draw two more circles, each of which is tangent to the original three
  - These come from Apollonius
- Continue drawing tangent circles



Koebe-Andreev-Thurston Theorem

For a finite, maximal planar graph G, there is a unique (up to circle inversions) circle packing whose tangency graph is isomorphic to G.

Koebe-Andreev-Thurston: An example



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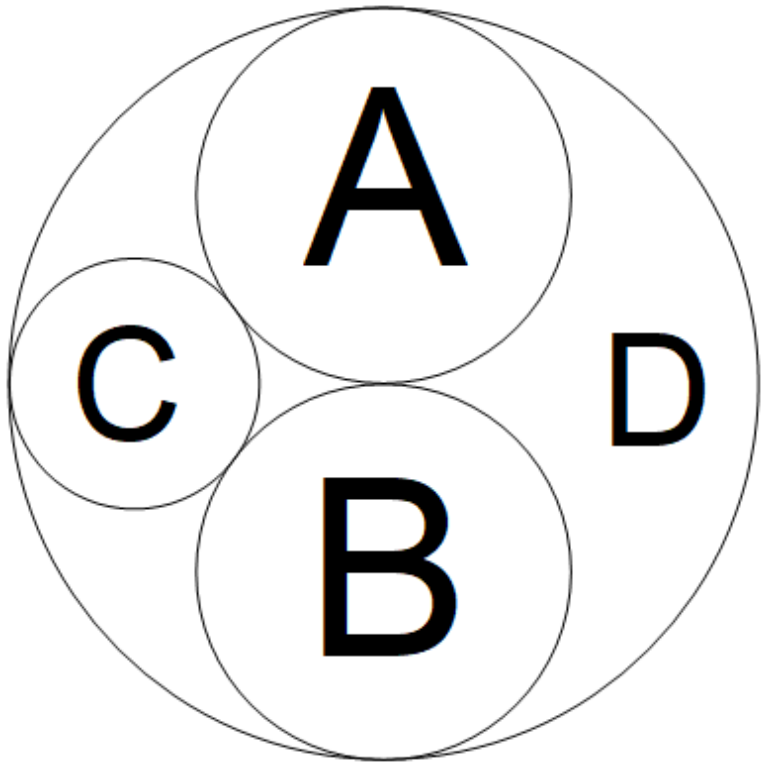

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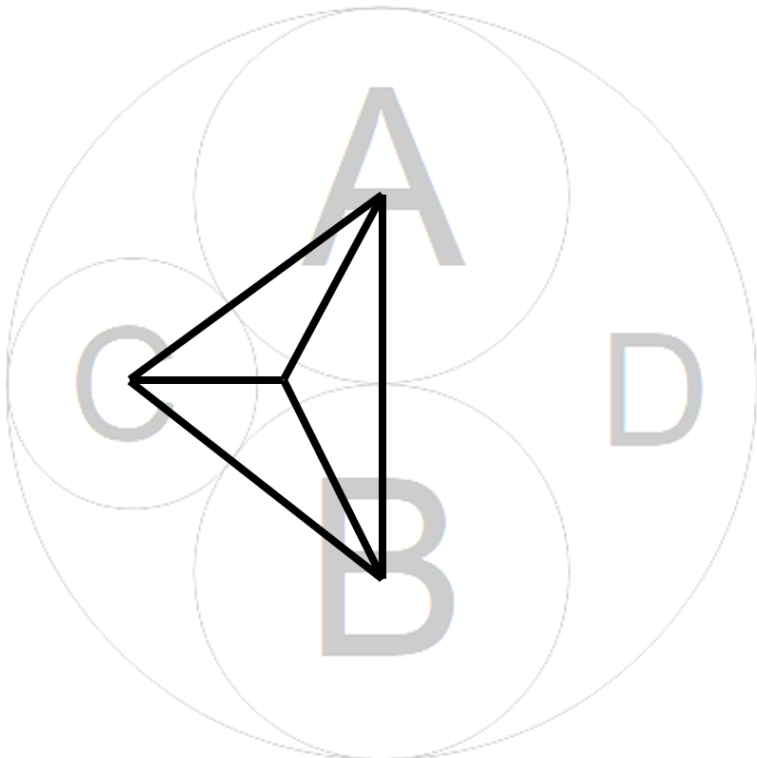
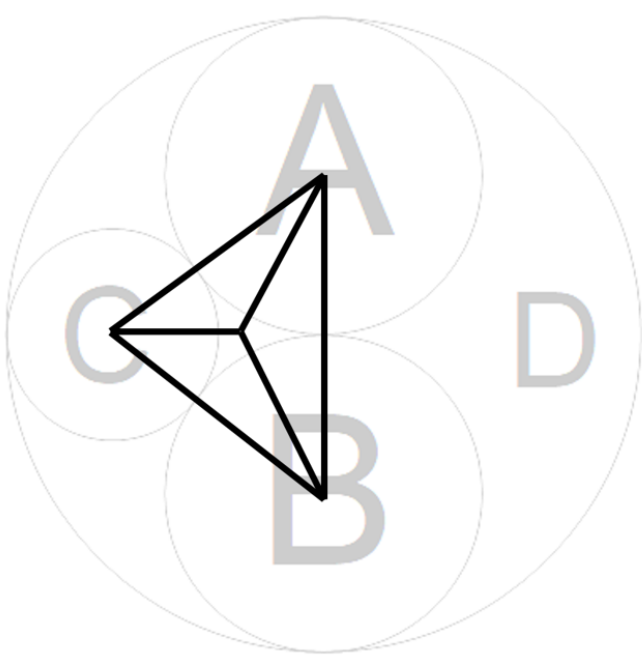
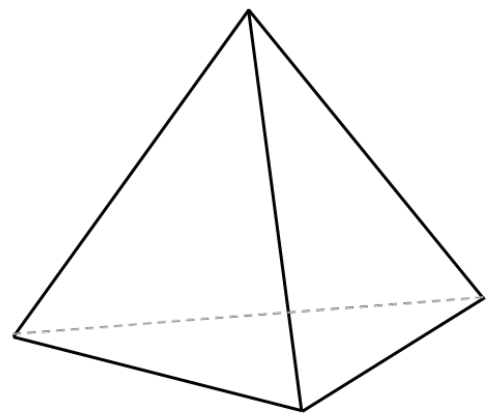
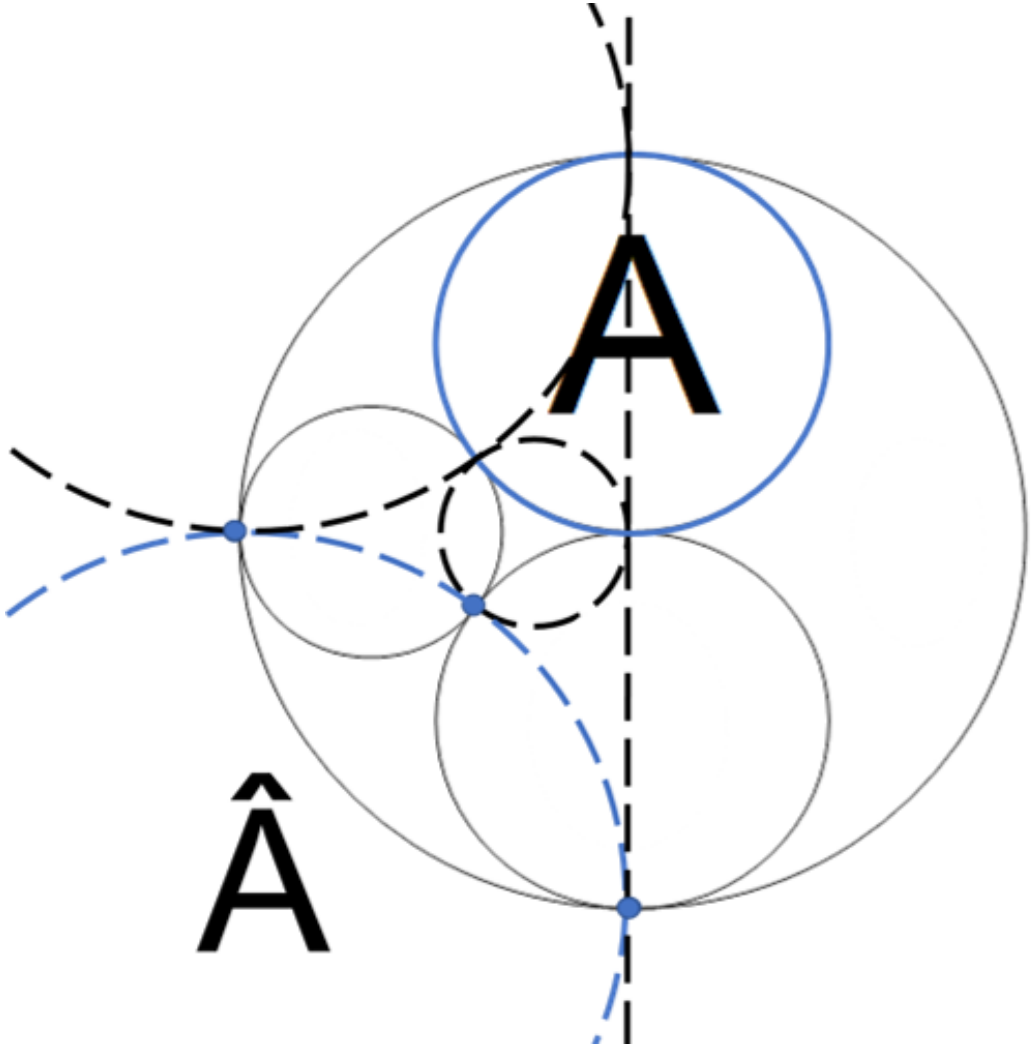


Koebe-Andreev-Thurston: An example





Dual Circles



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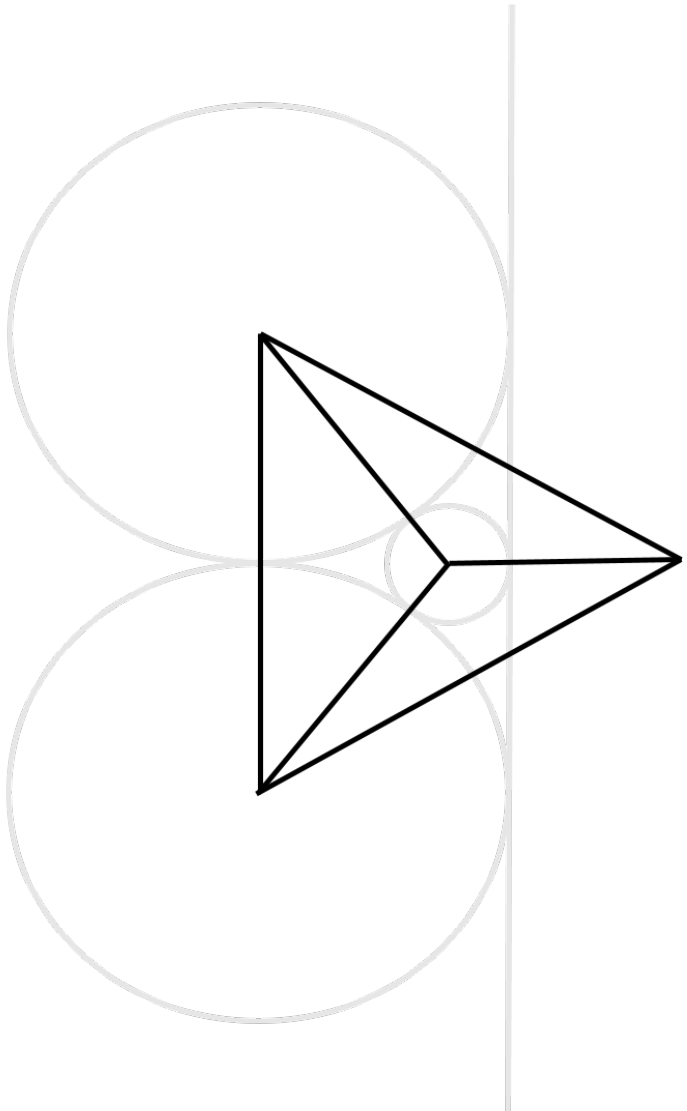


Dual Polyhedra

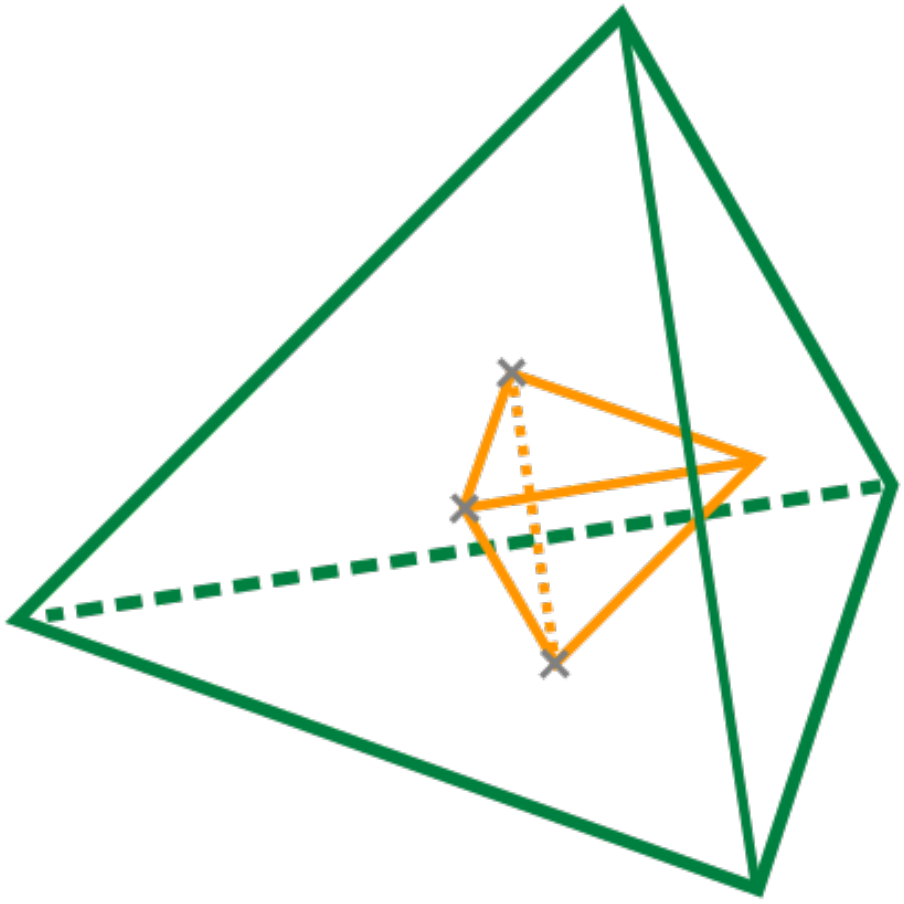
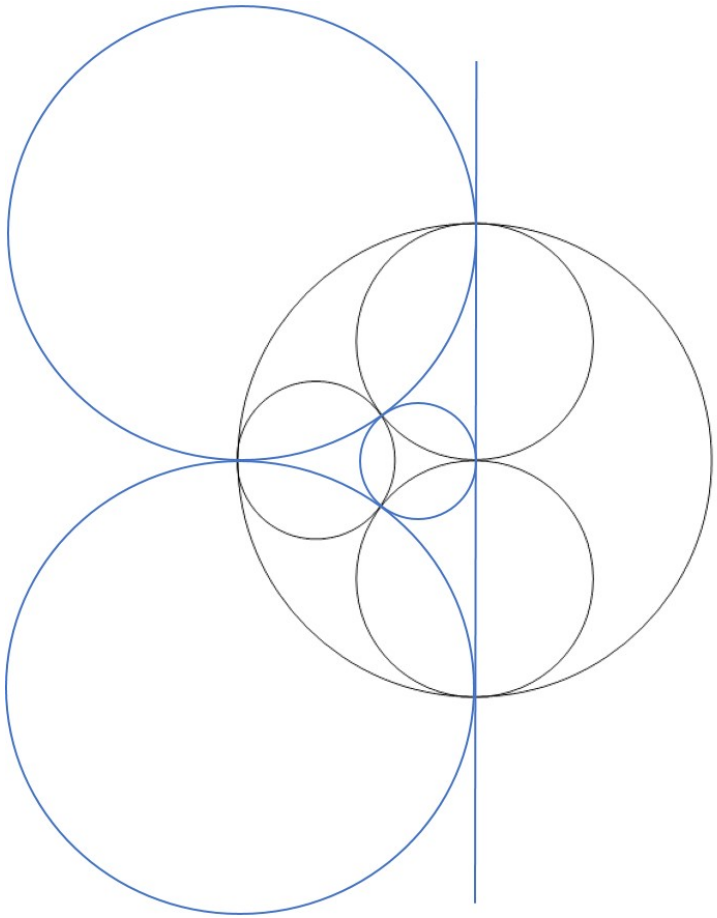
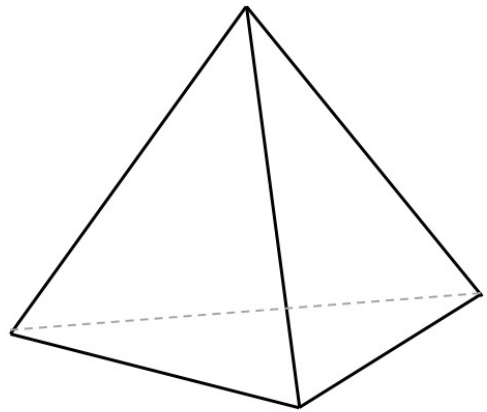
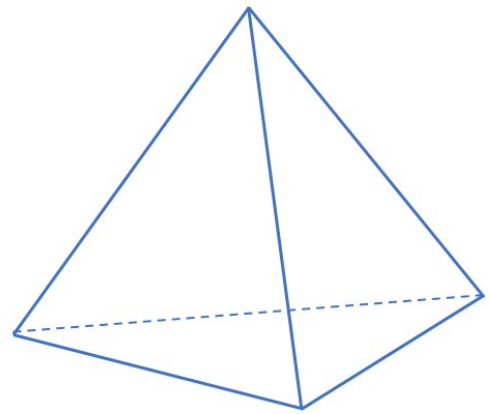


Figure: *Ekips39, Wikimedia Commons*

Dual Polyhedra





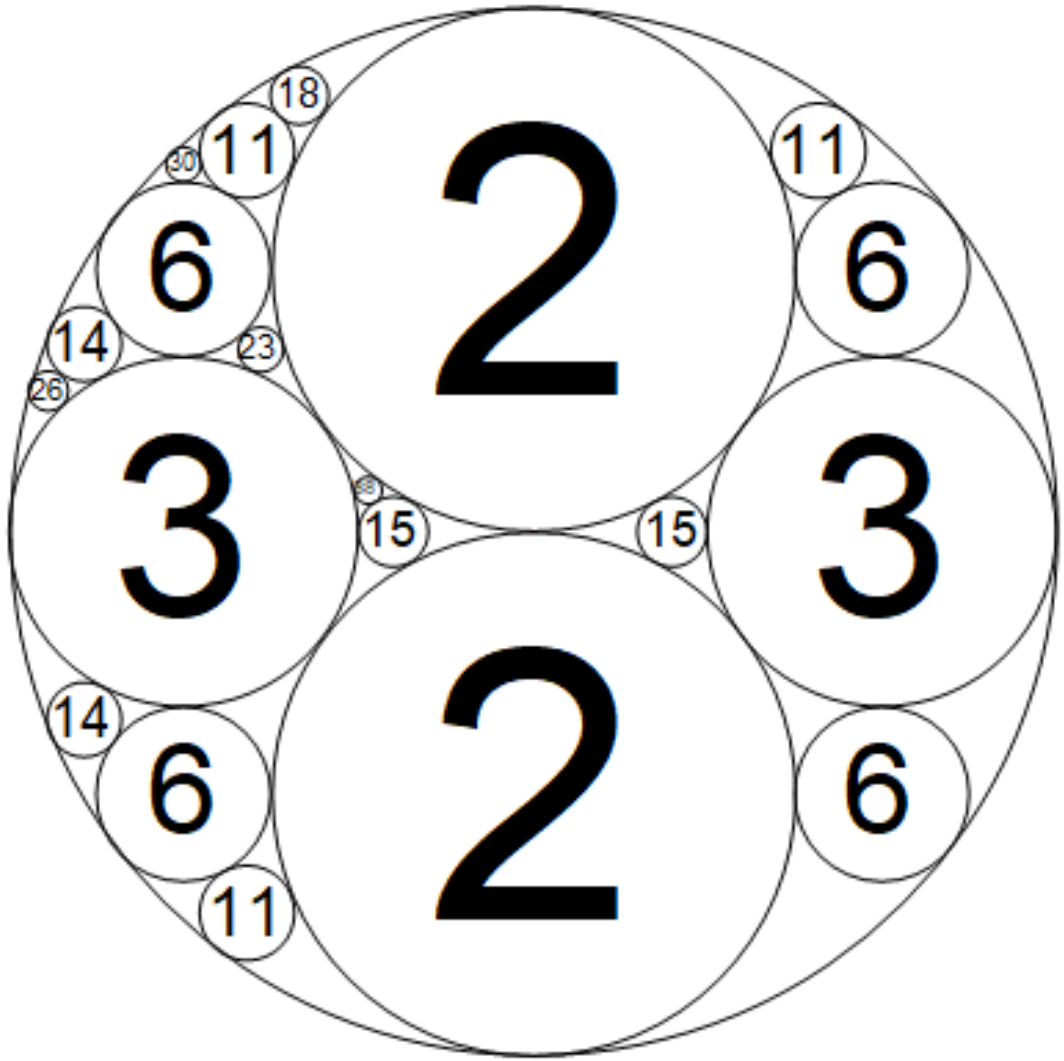


Integral Packings and Polyhedra

- We are interested in packings in which the *curvature* (reciprocal of the radius), generalized as *bend* in higher dimensions, is integral for every circle
  - From Descartes, Soddy found that if 4 mutually tangent circles have integer bends, then all circles in the packing have integer bends (true for Apollonian packings, but not in general)
- A polyhedron which has some associated integral circle packing is called an *integral polyhedron*
- Can we find and classify all integral polyhedra?

Tetrahedron

The Apollonian packing used as an example previously is integral, making the tetrahedron an integral polyhedron.



Approach

- How can we verify that a given packing is indeed integral?
  - This can be difficult, even with computers

Acknowledgments

Thanks to DIMACS, the Rutgers Math Department, the NSF, and Professor Kontorovich.